

ECON 257
EXERCISES 5

Classical linear model: Specification errors
Review questions

Consider a $T \times 1$ vector of observations y such that

$$y = X\beta + \varepsilon \quad (1)$$

where all the assumptions the classical linear model are satisfied.

1. Suppose that

$$y = X\beta + \varepsilon = X_1\beta_1 + x_k\beta_k + \varepsilon \quad (2)$$

where X_1 is a $T \times (k-1)$ fixed matrix and x_k is a $T \times 1$ fixed vector. Instead of (2), we estimate the incomplete model:

$$y = X_1\gamma + \varepsilon. \quad (3)$$

Discuss the properties of the least squares estimator $\hat{\gamma}$ based on the incomplete model (3) as an estimator of β_1 .

- (a) Find the expected value of $\hat{\gamma}$.
 - (b) Is $\hat{\gamma}$ an unbiased estimator of β_1 ?
 - (c) If $\hat{\gamma}$ is not an unbiased estimator of β_1 , give a condition under which it would be unbiased.
 - (d) Find the covariance matrix of $\hat{\gamma}$. Is it larger or smaller than the covariance matrix of $\hat{\beta}_1$ based on the complete model (2)?
 - (e) Discuss the properties of the estimator of σ^2 based on the incomplete model (3).
2. Discuss the consequence of having

$$E(\varepsilon) = \xi \neq 0 \quad (4)$$

on the properties of the least squares estimator of β in (1).

3. In model (1), suppose that

$$E(\varepsilon\varepsilon') = \Omega. \quad (5)$$

where Ω is known positive definite matrix.

- (a) Is the least squares estimator $\hat{\beta}$ unbiased ?
- (b) Find the covariance matrix $V(\hat{\beta})$.
- (c) Discuss the consequences of the above observations on testing the hypothesis $\beta_1 = 0$.
- (d) Propose a best linear unbiased estimator of β .