

**ECONOMETRICS 1**  
**EXERCISES 1**

**Covariances and covariance matrices**

1. Respond by TRUE, FALSE or UNCERTAIN to each one of the following statements and explain your answer (Maximum: 1 page per statement)

- (a) If a random variable  $X$  has zero variance, it is unpredictable.
- (b) If the correlation between two random variables  $X$  and  $Y$  is one, this means that one of them has variance zero.
- (c) If  $X$  is a random variable with zero variance, then its covariance with any other random variable  $Y$  cannot be negative.
- (d) If  $X$  is a random variable with variance equal to zero, then its correlation with any other random variable  $Y$  must be positive.
- (e) If  $X$  is a random variable with variance one, then its correlation with any other random variable  $Y$  must be positive.

2. Let  $X$  and  $Y$  be two random variables with finite variances  $\mathbb{V}(X)$  and  $\mathbb{V}(Y)$ . Prove the following inequality:

$$C(X, Y)^2 \leq \mathbb{V}(X) \mathbb{V}(Y). \quad (1)$$

3. Let  $\mathbf{X} = (X_1, \dots, X_k)'$  a  $k \times 1$  random vector,  $\alpha$  a scalar,  $\mathbf{a}$  and  $\mathbf{b}$  fixed  $k \times 1$  vectors, and  $A$  a fixed  $g \times k$  matrix. Then, provided the moments considered are finite, show that the following properties hold:

- (a)  $E(\mathbf{X} + \mathbf{a}) = E(\mathbf{X}) + \mathbf{a}$  ;
- (b)  $E(\alpha \mathbf{X}) = \alpha E(\mathbf{X})$  ;
- (c)  $E(\mathbf{a}'\mathbf{X}) = \mathbf{a}'E(\mathbf{X})$  ,  $E(A\mathbf{X}) = AE(\mathbf{X})$  ;
- (d)  $\mathbb{V}(\mathbf{X} + \mathbf{a}) = \mathbb{V}(\mathbf{X})$  ;
- (e)  $\mathbb{V}(\alpha \mathbf{X}) = \alpha^2 \mathbb{V}(\mathbf{X})$  ;
- (f)  $\mathbb{V}(\mathbf{a}'\mathbf{X}) = \mathbf{a}'\mathbb{V}(\mathbf{X})\mathbf{a}$  ,  $\mathbb{V}(A\mathbf{X}) = A\mathbb{V}(\mathbf{X})A'$  ;
- (g)  $C(\mathbf{a}'\mathbf{X}, \mathbf{b}'\mathbf{X}) = \mathbf{a}'\mathbb{V}(\mathbf{X})\mathbf{b} = \mathbf{b}'\mathbb{V}(\mathbf{X})\mathbf{a}$  .

4. Let  $\mathbf{X} = (X_1, \dots, X_k)'$  be a random vector with finite second moments and let  $\Sigma = \mathbb{V}(\mathbf{X})$  be its covariance matrix. Prove the following properties.

- (a)  $\Sigma' = \Sigma$ .
- (b)  $\Sigma$  is a positive semidefinite matrix.