Jean-Marie Dufour
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ECONOMETRICS 1
EXERCISES 2

Regression and prediction

1. Let X a random variable that follows a $N(0,1)$ distribution.

   (a) Find the best predictor of X^2 (in the mean square sense) based on X, and compute the variance of the corresponding prediction error.

   (b) Find the best linear prediction of X^2 (in the mean square sense) based on X, and compute the variance of the corresponding prediction error.

   (c) Find the best linear prediction of X (in the mean square sense) based on X^2, and compute the variance of the corresponding prediction error.

2. Let X a random variable that follows a $N(1,4)$ distribution.

   (a) Find the best prediction of X^2 (in the mean square sense) based on X, and compute the variance of the corresponding prediction error.

   (b) Find the best linear prediction of X^2 (in the mean square sense) based on X, and compute the variance of the corresponding prediction error.

   (c) Find the best linear prediction of X (in the mean square sense) based on X^2, and compute the variance of the corresponding prediction error.

3. Let $Z = (Y, X)'$ be a two-dimensional random vector such that

$$\mathsf{E}(Y) = E(X) = 1\,, \tag{1}$$

$$\mathsf{V}(Z) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}. \tag{2}$$

   (a) Build the best linear prediction of Y based on X (in the mean square sense).

   (b) Graph this linear prediction as a function of X.

   (c) What is the variance of the corresponding prediction error?

   (d) For $X = -1,\, 0,\, 1$, compute the best linear predictions of Y.

4. Let $Y,\ X_1,\, \dots,\, X_k$ be real random variables in L^2, and $X = (X_1,\, \dots,\, X_k)'$.

(a) What is the effect of changing the mean of Y on the conditional variance of Y given X ? Justify your answer.

(b) What is the effect of changing the means of Y and X on the variance of the best linear prediction Y based on X ? Justify your answer.