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ECONOMETRIC THEORY
REVIEW QUESTIONS

Multivariate dynamic models

1. Discuss the relationships between the following concepts:
   (a) Granger causality and prediction;
   (b) Granger causality and causality at several horizons;
   (c) Granger causality and impulse response coefficients;
   (d) causality at several horizons and impulse response coefficients.

2. Consider the process described by the following model:

$$X_t = \begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix} = \begin{bmatrix} 1-0.5B & 0 \\ -0.5B & 1-0.2B \end{bmatrix} \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} \tag{1}$$

   où $t \in \mathbb{Z}$, $a_t = [a_{1t},\ a_{2t}]'$ is a sequence of *i.i.d.* $N[0,\ \Sigma]$ random vectors with

$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} . \tag{2}$$

   (a) What is the type of this process?
   (b) Is this process strictly stationary? Why?
   (c) Does this process have a Wold representation? If so, give it.
   (d) Is this process invertible? Why?
   (e) Does this process has an autoregressive representation? If so, give it.
   (f) Does the variable X_{2t} cause X_{1t} in the sense of Granger ? Justify your answer.
   (g) Does the variable X_{1t} cause X_{2t} in the sense of Granger ? Justify your answer.
   (h) Does the variable X_{2t} cause X_{1t} at all horizons ? Justify your answer.
   (i) Does the variable X_{1t} cause X_{2t} at all horizons ? Justify your answer.