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ADVANCED ECONOMETRIC THEORY
EXERCISES 8

PREDICTION AND RESIDUALS

1. Exercise 11.1 in Gouriéroux and Monfort (1995, chap. 11).

2. Exercise 11.7 in Gouriéroux and Monfort (1995, chap. 11).

3. Let
$$Y_i = x_i'\beta + u_i\,, \quad i = 1,\, \dots\,,\, n+1$$
where all the hypotheses of the classical linear model are satisfied. If $\hat{\beta}_n$ is the ordinary least squares estimator of β based on $Y_1, ..., Y_n$, show that
$$\hat{Y}_{n+1} = x_{n+1}'\hat{\beta}_n \text{ and } \hat{e}_{n+1} = Y_{n+1} - \hat{Y}_{n+1}$$
are correlated.

4. Explain the difference between:
   (a) generalized residuals,
   (b) simulated residuals;
   (c) two-stage simulated residuals.

5. Consider the nonlinear regression model:
$$\begin{aligned} Y_t &= f\,(x_t;\theta) + u_t\,, \quad t = 1,\, \dots\,,\, T+h,\ h \geq 1 \\ u_t &= \rho u_{t-1} + \varepsilon_t\,,\ |\rho| < 1 \end{aligned}$$
where x_t is fixed and ε_t is a random disturbance independent of$u_{t-1}, u_{t-2}, ...$, such that $\mathsf{E}\,(\varepsilon_t) = 0$.
   (a) Assuming that θ and ρ are known, compute the prediction of Y_{t+h} (in the mean-square error sense) based on $Y_1, ..., Y_T$.
   (b) If θ and ρ are unknown, how would you predict Y_{t+h}?

6. Consider a PROBIT model with the latent variable

$$Y_i^* = x_i'\theta + u_i\,, \quad i = 1,\,\dots\,,\,n\,,$$

where $x_1,\,\dots\,,\,x_n$ are fixed and $u_1,\,\dots\,,\,u_n$ are independent $N\,(0,\,1)$ random variables. Find the generalized residuals for this model.

References

GOURIÉROUX, C., AND A. MONFORT (1995): *Statistics and Econometric Models, Volumes One and Two*. Cambridge University Press, Cambridge, U.K.