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ECONOMETRIC THEORY
EXERCISES 4

ESTIMATION THEORY

Reference: Gouriéroux and Monfort (1995, Chapter 5)

1. Consider the parametric model $(\mathscr{Y}, \mathscr{P})$, where $\mathscr{P} = \{P_\theta : \theta \in \Theta \subseteq \mathbb{R}^p\}$, and $m_y(d)$ a randomized estimator of $g(\theta)$. Let $L(d, \theta)$ be a convex loss function for the estimator $d \in g(\Theta)$.

   (a) Show there exists a nonrandomized estimator $\delta(y)$, where $y \in \mathscr{Y}$, which is preferable to $m_y(d)$.

   (b) Let $S(y)$ be a sufficient statistic for θ. Show the estimator $\delta(y)$ can be improved by an estimator which is a function of $S(y)$ only.

2. Let $g(\theta)$ be a parameter in $\mathbb{R}^q$ and $d(y)$ a (nonrandomized) estimator of $g(\theta)$.

   (a) Define the matrix quadratic loss function for the estimation of $g(\theta)$ by $d(y)$.

   (b) Show that an optimal estimator with respect to the quadratic loss function also minimizes the scalar risk

   $$L_c(d, \theta) = \left(c'[d - g(\theta)]\right)^2 \text{ for all } c \in \mathbb{R}^q.$$

3. Let Y_1 and Y_2 be two independent observations from a Poisson distribution $P(\lambda)$. Consider the two estimators

   $$\begin{aligned} \delta_1(Y) &= (Y_1 + Y_2)\,/\,2, \\ \delta_2(Y) &= [Y_1 - \delta_1(Y)]^2 + [Y_2 - \delta_1(Y)]^2, \end{aligned}$$

   where $Y = (Y_1, Y_2)'$.

   (a) i. Is the estimator $\delta_1(Y)$ unbiased? Justify your answer.

   ii. Is the estimator $\delta_2(Y)$ unbiased? Justify your answer.

(b) Show that $\delta_1(Y)$ is preferable to $\delta_2(Y)$ with respect to quadratic loss.

4. Define the four following concepts:

    (a) asymptotically unbiased estimator;
    (b) weakly consistent estimator;
    (c) consistent estimator in quadratic mean;
    (d) strongly consistent estimator.

5. Show that an estimator which is consistent in quadratic mean is:

    (a) asymptotically unbiased;
    (b) weakly consistent.

6. Exercise 5.1 in Gouriéroux and Monfort (1995, chap. 5).

7. Exercise 5.3 in Gouriéroux and Monfort (1995, chap. 5).

8. Exercise 5.6 in Gouriéroux and Monfort (1995, chap. 5).

References

GOURIÉROUX, C., AND A. MONFORT (1995): *Statistics and Econometric Models, Volumes One and Two*. Cambridge University Press, Cambridge, U.K., Translated by Quang Vuong.