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ECONOMETRIC THEORY
EXERCICES 3
INFORMATION

Reference: Gouriéroux and Monfort (1995, Chapter 3)

1. Define the following notions:
   (a) sufficient statistic;
   (b) ancillary statistic;
   (c) Fisher information;
   (d) complete statistic.
2. Let $Y_1, \ldots, Y_n$ be independent and identically distributed random variables with the same density $f(y; \theta)$. Show that the order statistics are sufficient for θ.
3. If the random variables $Y_1, \ldots, Y_n$ are independent $N(0, \sigma^2)$, find a sufficient statistic for σ^2.
4. State and demonstrate the factorization criterion for a sufficient statistic.
5. What are the sufficient statistics for an exponential model? Are these statistics minimal? Justify your answers.
6. Let $\ell(Y; \theta)$ be the likelihood function for the sample $Y = (Y_1, \ldots, Y_n)'$. Show that

$$I(\theta) = E\left[-\frac{\partial^2 \log \ell(Y; \theta)}{\partial \theta \, \partial \theta'}\right].$$

7. When is parameter
   (a) identifiable?
   (b) locally identifiable?
8. When is a parametric model identifiable?

References

GOURIÉROUX, C., AND A. MONFORT (1995): *Statistics and Econometric Models, Volumes One and Two*. Cambridge University Press, Cambridge, U.K., Translated by Quang Vuong.