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March 2010

Compiled: April 5, 2010

ADVANCED ECONOMETRIC THEORY
EXERCISES 12
METHODS OF MOMENTS

1. a) Explain the method of asymptotic least squares.

   b) Under appropriate regularity conditions, describe the asymptotic distribution of the asymptotic least squares estimator.

2. Let $Y_1, \ldots, Y_n$ a sample of i.i.d. observations from the exponential family

$$f(y, \theta) = C(\theta)\, h(y) \exp\left[Q(\theta)\, T(y)\right] .$$

   Describe how one could use the method of asymptotic least squares to estimate θ.

3. We have repeated observations $Y_{ik},\ i = 1, \ldots, n,\ k = 1, \ldots, K$ of qualitative dichotomic variables, under K different conditions $x_k = (x_{k1}, \ldots, x_{kp})$, $k = 1, \ldots, K$, with $K \geq p$. These observations are independent such that

$$\begin{aligned} P[Y_{ik} = 1] &= \frac{1}{1 + \exp(-x_k \theta)} \equiv p_k(\theta) , \\ P[Y_{ik} = 0] &= 1 - p_k(\theta) , \end{aligned}$$

   where $\theta = (\theta_1, \ldots, \theta_p)'$ is a parameter of dimension p.

   a) Write the likelihood function associated with this sample and show that it belongs to the exponential family.

   b) Describe the method of Berkson to estimate θ. Can the Berkson estimator be viewed as an asymptotic least squares estimator? Explain why.

4. Exercise 9.3 in Gouriéroux and Monfort (1995, Cahpter 9).

5. Let the linear model

$$Y_i = \sum_{i=1}^{k} X_{ik}\, b_{0k} + u_i, i = 1, \ldots, n$$

where $b_0 = (b_{01}, \ldots, b_{0k})'$ is vector of fixed parameters and the vectors

$$(u_i, X_{i1}, \ldots, X_{ik}, Z_{i1}, \ldots, Z_{iH})', i = 1, \ldots, n$$

are independent with finite second moments. Further, the variables $Z_i = (Z_{i1}, \ldots, Z_{iH})'$ represent instrumental variables such that

$$E\left(u_i \mid Z_i\right) = 0, V\left(u_i \mid Z_i\right) = \sigma_0^2,\; i = 1, \ldots, n\,.$$

   a) Describe the instrumental variable estimator $b_{IV}(A)$ of b_0 based on ZA, where A is a selection matrix and $Z' = [Z_1, \ldots, Z_n]$.

   b) Find the asymptotic distribution of $b_{IV}(A)$. (If necessary, add the required regularity conditions.)

   c) Show that an optimal choice for A is obtained by taking

$$A^* = E\left(Z'Z\right)^{-1} E\left(Z'X\right)\,.$$

6. a) Describe the generalized method of moments.

   b) Describe how an optimal generalized method of moments estimator can be obtained.

Reference: Gouriéroux and Monfort (1995, Chapter 9).

References

Gouriéroux, C., and A. Monfort (1995): *Statistics and Econometric Models, Volumes One and Two*. Cambridge University Press, Cambridge, U.K.