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ADVANCED ECONOMETRIC THEORY
EXERCISES 11

TESTS BASED ON LIKELIHOOD FUNCTIONS

1. Explain in an intuitive way the principles underlying the following tests: Wald test, Rao score test, likelihood ratio, Neyman's $C(\alpha)$, and Hausman test.

2. Let (Y_i, X_i) , $i = 1, \ldots, n$, be observations such that the conditional likelihood of $Y = (Y_1, \ldots, Y_n)$ given $X = (X_1, \ldots, X_n)'$ has the form:
$$L_n(\theta) = \prod_{i=1}^{n} f(y_i \mid x_i, \theta)$$
where θ is a $p \times 1$ parameter vector. Further, suppose the regularity conditions of Property 7.17 of Gouriéroux and Monfort (1995, Chapter 7, Section 7.4) are satisfied. We consider an implicit hypothesis $H_0 : g(\theta) = 0$, where $g(\theta)$ is an $r \times 1$ vector such that the matrix $\partial g/\partial\theta'$ has rank r $(1 \leq r \leq p)$.
   (a) Derive the asymptotic distribution (under H_0) of the Wald statistic for testing H_0.
   (b) Show that this test is consistent.

3. Under the conditions of question 2,
   (a) describe Rao's score statistic for testing H_0, and show it is identical to the Lagrange multiplier statistic;
   (b) derive the asymptotic distribution of the score statistic;
   (c) show that the score statistic is asymptotically equivalent to the Wald statistic (under H_0);
   (d) give a sufficient condition under which the score test is consistent.

4. Under the conditions of question 2, show that the Wald, Rao score and likelihood ratio statistics are asymptotically equivalent under H_0.

5. Under the conditions of question 2, for an hypothesis of the type $H_0 : \theta_1 = \theta_1^0$, where $\theta = (\theta_1', \theta_2')'$ and θ_i is a $p_i \times 1$ vector $(i = 1, 2)$,

   (a) describe Neyman's $C(\alpha)$ statistic for testing H_0;

   (b) show that this statistic is asymptotically equivalent (under H_0) to the Wald statistic, and derive the asymptotic of the $C(\alpha)$ statistic under H_0;

   (c) describe how a $C(\alpha)$ test can be implemented through an artificial regression.

Reference: Gouriéroux and Monfort (1995, Chapter 17).

References

GOURIÉROUX, C., AND A. MONFORT (1995): *Statistics and Econometric Models, Volumes One and Two*. Cambridge University Press, Cambridge, U.K.