Finite-sample inference in econometrics and statistics *

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First version: December 1998
This version: November 29, 2006, 4:04pm

* This work was supported by the Canada Research Chair Program (Chair in Econometrics, Université de Montréal), the Canadian Network of Centres of Excellence [program on *Mathematics of Information Technology and Complex Systems* (MITACS)], the Canada Council for the Arts (Killam Fellowship), the Natural Sciences and Engineering Research Council of Canada, the Social Sciences and Humanities Research Council of Canada, and the Fonds FCAR (Government of Québec).

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Contents

1. Hypothesis testing and nuisance parameters

Testing an hypothesis H_0 usually involves finding a test statistic $T(H_0)$ with 2 characteristics:

1. the stochastic behavior (distribution) of $T(H_0)$ under H_0 must be known;
2. the way the distribution of $T(H_0)$ is affected under the alternative must also be known [e.g. $T(H_0)$ may tend to take **large** or **small** values with greater possibilities under the alternative].

$\longrightarrow$ Fundamental that the quantiles of the distribution function of $T(H_0)$ be either **uniquely defined** or (at least) **bounded.**
Otherwise, the behavior of $T(H_0)$ under H_0 is **not interpretable** and $T(H_0)$ **cannot** be the basis of a **valid** test of H_0.

Common difficulty: nuisance parameters

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \tag{1.1}$$

$$H_0 : \theta_1 = \theta_1^0 \tag{1.2}$$

$$\text{Test: } T\left(\theta_1^0\right) > c(\alpha) \tag{1.3}$$

2 basic cases:

1. the distribution of $T(\theta_1^0)$ does not depend on θ_2: it is uniquely determined;
2. the distribution of $T(\theta_1^0)$ depends on θ_2 : it is not uniquely determined:
   θ_2 is a **nuisance parameter**.

In many econometric and statistical problems, it is difficult to find the exact distribution of test statistics and confidence sets.
Two basic reasons:

1. deriving the relevant distributions may require complex calculations
   (even when there is no nuisance parameter);

   and / or
2. distribution may involve nuisance parameters.

Most common approach to such distributional problems: use a **large-sample approximation.**

Important characteristic of such approximations in many situations, the asymptotic distribution does not involve nuisance parameters [e.g., N(0,1), chi-square]
$\longrightarrow$ great flexibility.

Main interest of asymptotic approximations: generate approximations useful in finite-samples

Shortcomings:

1. Finite-sample distribution may involve nuisance parameters
2. Accuracy of the approximation is typically unknown and may be **arbitrarily bad** especially with nuisance parameters (non-uniform convergence)

Approximation arbitrarily bad $\Longrightarrow$ Tests statistic not interpretable

2. Distributions without nuisance parameters

Deriving analytically the distribution of a test statistic typically involves complex calculations and remain feasible only in special cases.
t and Fdistributions in the classical linear model.

Nowadays, it is often possible to simulate the relevant test statistic under the null hypothesis.

1. The distribution of the test statistic – and thus the relevant critical values – can be evaluated to any degree of precision (with a sufficiently **large** number of replications).
2. A Monte Carlo (MC) test can performed:
   the size of the test can be perfectly controlled, even with a **small** number replications [Dwass (1957), Barnard (1963)].

Work exploiting the technique of MC tests in econometrics:

- Dufour and Kiviet (1996, Journal of Econometrics)
- Kiviet and Dufour (1997, Journal of Econometrics)
- Dufour and Kiviet (1998, Econometrica)
- Dufour, Farhat, Gardiol, and Khalaf (1998, Econometrics Journal)
- Dufour and Khalaf (2001, Baltagi, eds, Blackwell)

Dufour and Khalaf (2002, Journal of Econometrics)

- Dufour, Khalaf, and Beaulieu (2003, Oxford Bulletin of Economics and Statistics)
- Dufour, Khalaf, Bernard, and Genest (2004, Journal of Econometrics)
- Dufour (2006, Journal of Econometrics): generalized theory of MC tests
- Beaulieu, Dufour, and Khalaf (2006, Journal of Business and Economic Statistics, forth.)

3. Basic techniques to deal with nuisance parameters

1. **Transforming**:
   Find a transformation that reduces the data for a statistic $T(\theta_1^0)$ whose distribution does not depend on θ_2 [e.g. reduction to a maximal invariant statistic]

   - t and F-statistics in classical linear regression
   - reduction of observations in cash or signs

2. **Conditioning**
   on a statistic S such that the conditional distribution of $T(\theta_1^0)$ given S does not depend on θ_2:

   - conditioning on explanatory variables.
   - tests with Neyman structure;
   - permutation tests;

3. **Bounding**:
   find a bound on the distribution of $T(\theta_1^0)$ which is valid irrespective of the unknown value of θ_2:

$$\sup_{\theta_2} P_{(\theta_1^0,\theta_2)}\left[T(\theta_1^0) > x\right] \;\leq\; B_{\theta_1^0}(x)$$
$$\inf_{\theta_2} P_{(\theta_1^0,\theta_2)}\left[T(\theta_1^0) > x\right] \;\geq\; C_{\theta_1^0}(x)$$

4. Approaches for building bounds procedures

Four approaches:

1. Bounding the statistics of interest by other random variables with more tractable distributions
   $\longrightarrow$ Bounds on distribution functions
2. Bounding directly the distribution function of interest (or its tail areas) by some function (not necessarily obtained as the distribution function of random variable)
3. Sequential confidence procedures
4. Projection techniques

4.1. Bounding the statistic of interest by other statistics

Given a statistic T used in building a test on confidence set with a complicated distribution (possibly involving nuisance parameters), one tries to find other statistics T_1 and T_2 with more tractable distributions and such that

$$\begin{aligned} T_1 &\leq T \leq T_2 \,, \\ P\left[T_1 \geq x\right] &\leq P\left[T \geq x\right] \leq P\left[T_2 \geq x\right] \,. \end{aligned}$$

Approach applied in:

- Dufour (1989, Econometrica)
- Dufour (1990, Econometrica)
- Dufour and Khalaf (2002, Journal of Econometrics)
- Dufour (2006, Journal of Econometrics): Maximized Monte Carlo (MMC) tests

Dufour and Jouini (2006, Journal of Econometrics): MMC tests applied to VAR models

4.2. Bounding tail areas by some function

$$P_\theta [T \geq x] \leq G(x)$$

where $G(x)$ is not necessarily obtained from the distribution of a random variables.

1. Exponential inequalities;
2. Chebyshev inequalities (based on second and higher-order moments);
3. Berry-Esséen bounds.

There are cases (e.g. in nonparametric statistics) where such bounds can be used and combined to get fairly tight bounds on tail areas.

Approach used in:

- Dufour (1991, Hackl-Westlund, eds., Springer)
- Dufour and Mahseredjian (1993, Econometric Theory)
- Dufour and Hallin (1991, Econometric Theory)
- Dufour and Hallin (1992a, Econometric Theory)
- Dufour and Hallin (1992b, Journal of Statistical Planning and Inference)
- Dufour and Hallin (1993, JASA)

4.3. Projection techniques

Let

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} . \tag{4.1}$$

Very often, it is easy to relatively easy to find a joint confidence set for θ :

$$\mathsf{P}[\theta \in C_\theta(\alpha)] \geq 1 - \alpha \, , \tag{4.2}$$

then, for any function $g(\theta)$,

$$\mathsf{P}\Big[g(\theta) \in g\left[C_\theta(\alpha)\right]\Big] \geq 1 - \alpha \, . \tag{4.3}$$

$g\left[C_\theta(\alpha)\right]$ is a confidence set with level $1-\alpha$ for $g(\theta)$. For example, we can take

$$g(\theta) = \theta_2 \, . \tag{4.4}$$

If θ_2 is a scalar and $C_\theta(\alpha)$ is a compact set, then $g\left[C_\theta(\alpha)\right]$ must be an interval.

Approach applied in:

- Dufour (1990, Ecnometrica)
- Dufour (1997, Ecnometrica)
- Abdelkhalek and Dufour (1998, Review of Economics and Statistics)
- Dufour, Hallin, and Mizera (1998, Journal of Nonparametric Statistics)
- Dufour and Kiviet (1998, Econometrica)
- Dufour and Jasiak (2001, International Economic Review)
- Dufour and Khalaf (2002)
- Dufour and Taamouti (2005, Econometrica)
- Dufour and Taamouti (2006, Journal of Econometrics, forth.)

4.4. Sequential confidence procedure

Useful with nuisance parameters

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \tag{4.5}$$

where

$$\begin{aligned} &\theta_1\text{: vector of nuisance parameter} \\ &\theta_2\text{: vector of parameters of interest} \end{aligned} \tag{4.6}$$

Problem: inference about θ_2 (confidence set on test)

Suppose 2 conditions are satisfied:

1. it is possible to build exact confidence set C_1 for θ_1
$$P\left[\theta_1 \in C_1\right] = 1 - \alpha_1\,; \tag{4.7}$$
2. if θ_1 is known, it is possible to build a confidence set $C_2(\theta_1)$ for θ_2 such that
$$P[\theta_2 \in C_2(\theta_1)] = 1 - \alpha_2\,. \tag{4.8}$$

Procedure:

1. Build on exact confidence set C_1 for θ_1:
$$P\left[\theta_1 \in C_1\right] \underset{(\geq)}{=} 1 - \alpha_1$$

2. Build a simultaneous confidence set C for θ_1 and θ_2:

$$C = \{(\theta_1, \theta_2) : \theta_1 \in C_1 \;,\; \theta_2 \in C_2(\theta_1)\}$$
$$P\left[(\theta_1, \theta_2) \in C\right] \geq 1 - (\alpha_1 + \alpha_2)$$

3. Use a projection (or an intersection) method to deduce conservative (or a liberal) confidence set for θ_2:

$$U = \{\theta_2 : (\theta_1, \theta_2) \in C \text{ for some } \theta_1 \in C_1\}$$
$$P\left[\theta_2 \in U\right] \geq 1 - (\alpha_1 + \alpha_2)$$

$$L = \{\theta_2 : (\theta_1, \theta_2) \in C \text{ for all } \; \theta_1 \in C_1\} \quad (4.9)$$
$$P\left[\theta_2 \in L\right] \leq (1 - \alpha_2) + \alpha_1 \quad (4.10)$$

4. Conservative and liberal critical regions can be deduced from there confidence sets:
$\theta_2^0 \notin U$ is a conservative critical region for H_0:$\theta_2 = \theta_2^0$ with level $\alpha \equiv \alpha_1 + \alpha_2$;
$\theta_2 \notin L$ is a liberal continual region for $H_0 : \theta_2 = \theta_2^0$ with level $\alpha = \alpha_1 - \alpha_2$.

5. By combining a conservative and a liberal confidence region with the same level one gets a gen-

eralized bounds tests

Approach applied to linear regression with AR(1) errors in:

- Dufour (1990, Ecnometrica)
- Dufour, Hallin, and Mizera (1998, Journal of Nonparametric Statistics)
- Dufour and Kiviet (1998, Econometrica)
- Beaulieu, Dufour, and Khalaf (2006, Journal of Business and Economic Statistics, forth.)

5. Weak identification

Several authors in the pas have noted that usual asymptotic approximations are not valid or lead to very inaccurate results when parameters of interest are close to regions where these parameters are not anymore identifiable:

Sargan (1983, Econometrica)
Phillips (1984, International Economic Review)
Phillips (1985, International Economic Review)
Gleser and Hwang (1987, Annals of Statistics)
Koschat (1987, Annals of Statistics)
Phillips (1989, Econometric Theory)
Hillier (1990, Econometrica)
Nelson and Startz (1990a, Journal of Business)
Nelson and Startz (1990b, Econometrica)
Buse (1992, Econometrica)
Maddala and Jeong (1992, Econometrica)
Choi and Phillips (1992, Journal of Econometrics)
Bound, Jaeger, and Baker (1993, NBER Discussion Paper)
Dufour and Jasiak (1993, CRDE)
Bound, Jaeger, and Baker (1995, Journal of the

American Statistical Association)

McManus, Nankervis, and Savin (1994, Journal of Econometrics)

Hall, Rudebusch, and Wilcox (1996, International Economic Review)

Dufour (1997, Econometrica)

Shea (1997, Review of Economics and Statistics)

Staiger and Stock (1997, Econometrica)

Wang and Zivot (1998, Econometrica)

Zivot, Startz, and Nelson (1998, International Economic Review)

Startz, Nelson, and Zivot (1999, International Economic Review)

Perron (1999)

Stock and Wright (2000, Econometrica)

Dufour and Jasiak (2001, International Economic Review)

Dufour and Taamouti (2001)

Kleibergen (2001, 2002)

Moreira (2001, 2002)

Stock and Yogo (2002)

Stock, Wright, and Yogo (2002, Journal of Business and Economic Statistics)

Dufour (2003, Canadian Journal of Economics)

Dufour and Taamouti (2005, Econometrica)

Dufour and Taamouti (2006, Journal of Econometrics, forth.)

1. Theoretical results show that the distributions of various estimators depend in a complicated way upon unknown nuisance parameters. So they are difficult to interpret.
2. When identification conditions do not hold, standard asymptotic theory for estimators and test statistics typically collapses.
3. With weak instruments,
   (a) 2SLS becomes heavily biased (in the same direction as OLS),
   (b) distribution of 2SLS is quite far the normal distribution (e.g., bimodal).
4. Problems were strikingly illustrated by the reconsideration by Bound, Jaeger, and Baker (1995, Journal of the American Statistical Association) of a study on returns to education by Angrist and Krueger (1991, QJE):

   329000 observations;

   replacing the instruments used by Angrist and Krueger (1991, QJE) with randomly generated instruments (totally irrelevant) produced very similar point estimates and standard errors;

indicates that the instruments originally used were weak.

Crucial to use finite-sample approaches to produce reliable inference.

Finite-sample approaches to inference on models involving weak identification

- Dufour (1997, Econometrica)
- Dufour and Jasiak (2001, International Economic Review)
- Dufour and Taamouti (2005, Econometrica)
- Dufour and Taamouti (2006, Journal of Econometrics, forth.)

Applications

1. Tobin's q
   [Dufour and Jasiak (2001, International Economic Review)]
2. Students' achievements and self-esteem
   [Dufour and Jasiak (2001, International Economic Review)]
3. Education and earnings
   [Dufour and Taamouti (2006, Journal of Econometrics, forth.)]
4. Trade and growth
   [Dufour and Taamouti (2006, Journal of Econometrics, forth.)]
5. New Keynesian Phillips curves
   [Dufour, Khalaf, and Kichian (2006a, Journal of Economic Dynamics and Control), Dufour, Khalaf, and Kichian (2006b)]
6. Black's CAPM
   [Beaulieu, Dufour, and Khalaf (2005)]

References

ABDELKHALEK, T., AND J.-M. DUFOUR (1998): "Statistical Inference for Computable General Equilibrium Models, with Application to a Model of the Moroccan Economy," *Review of Economics and Statistics*, LXXX, 520–534.

ANGRIST, J. D., AND A. B. KRUEGER (1991): "Does Compulsory School Attendance Affect Schooling and Earning?," *Quarterly Journal of Economics*, CVI, 979–1014.

BARNARD, G. A. (1963): "Comment on 'The Spectral Analysis of Point Processes' by M. S. Bartlett," *Journal of the Royal Statistical Society, Series B*, 25, 294.

BEAULIEU, M.-C., J.-M. DUFOUR, AND L. KHALAF (2005): "Testing Black's CAPM with Possibly Non-Gaussian Errors: An Exact Identification-Robust Simulation-Based Approach," Discussion paper, Centre interuniversitaire de recherche en analyse des organisations (CIRANO) and Centre interuniversitaire de recherche en économie quantitative (CIREQ), Université de Montréal.

——— (2006): "Multivariate Tests of Mean-Variance Efficiency with Possibly Non-Gaussian Errors: An Exact Simulation-Based Approach," *Journal of Business and Economic Statistics*, forthcoming.

BOUND, J., D. A. JAEGER, AND R. BAKER (1993): "The Cure can be Worse than the Disease: A Cautionary Tale Regarding Instrumental Variables," Technical Working Paper 137, National Bureau of Economic Research, Cambridge, MA.

BOUND, J., D. A. JAEGER, AND R. M. BAKER (1995): "Problems With Instrumental Variables Estimation When the Correlation Between the Instruments and the Endogenous Explanatory Variable Is Weak," *Journal of the American Statistical Association*, 90, 443–450.

BUSE, A. (1992): "The Bias of Instrumental Variables Estimators," *Econometrica*, 60, 173–180.

CHOI, I., AND P. C. B. PHILLIPS (1992): "Asymptotic and Finite Sample Distribution Theory for IV Estimators and Tests in Partially Identified Structural Equations," *Journal of Econometrics*, 51, 113–150.

DUFOUR, J.-M. (1989): "Nonlinear Hypotheses, Inequality Restrictions, and Non-Nested Hypotheses: Exact Simultaneous Tests in Linear Regressions," *Econometrica*, 57, 335–355.

——— (1990): "Exact Tests and Confidence Sets in Linear Regressions with Autocorrelated Errors," *Econometrica*, 58, 475–494.

——— (1991): "Kimball's Inequality and Bounds Tests for Comparing Several Regressions under Heteroskedasticity," in *Economic Structural Change. Analysis and Forecasting*, ed. by P. Hackl, and A. Westlund, pp. 49–57. Springer-Verlag, Berlin.

——— (1997): "Some Impossibility Theorems in Econometrics, with Applications to Structural and Dynamic Models," *Econometrica*, 65, 1365–1389.

——— (2003): "Identification, Weak Instruments and Statistical Inference in Econometrics," *Canadian Journal of Economics*, 36(4), 767–808.

——— (2006): "Monte Carlo Tests with Nuisance Parameters: A General Approach to Finite-Sample Inference and Nonstandard Asymptotics in Econometrics," *Journal of Econometrics*, 133(2), 443–477.

DUFOUR, J.-M., A. FARHAT, L. GARDIOL, AND L. KHALAF (1998): "Simulation-Based Finite Sample Normality Tests in Linear Regressions," *The Econometrics Journal*, 1, 154–173.

DUFOUR, J.-M., AND M. HALLIN (1991): "Nonuniform Bounds for Nonparametric t Tests," *Econometric Theory*, 7, 253–263.

——— (1992a): "Improved Berry-Esseen-Chebyshev Bounds with Statistical Applications," *Econometric Theory*, 8, 223–240.

——— (1992b): "Simple Exact Bounds for Distributions of Linear Signed Rank Statistics," *Journal of Statistical Planning and Inference*, 31, 311–333.

——— (1993): "Improved Eaton Bounds for Linear Combinations of Bounded Random Variables, with Statistical Applications," *Journal of the American Statistical Association*, 88, 1026–1033.

DUFOUR, J.-M., M. HALLIN, AND I. MIZERA (1998): "Generalized Runs Tests for Heteroskedastic Time Series," *Journal of Nonparametric Statistics*, 9, 39–86.

DUFOUR, J.-M., AND J. JASIAK (1993): "Finite Sample Inference Methods for Simultaneous Equations and Models with Unobserved and Generated Regressors," Discussion paper, C.R.D.E., Université de Montréal, 38 pages.

DUFOUR, J.-M., AND J. JASIAK (2001): "Finite Sample Limited Information Inference Methods for Structural Equations and Models with Generated Regressors," *International Economic Review*, 42, 815–843.

DUFOUR, J.-M., AND T. JOUINI (2006): "Finite-Sample Simulation-Based Tests in VAR Models with Applications to Granger Causality Testing," *Journal of Econometrics*, 135(1-2), 229–254.

DUFOUR, J.-M., AND L. KHALAF (2001): "Monte Carlo Test Methods in Econometrics," in *Companion to Theoretical Econometrics*, ed. by B. Baltagi, Blackwell Companions to Contemporary Economics, chap. 23, pp. 494–519. Basil Blackwell, Oxford, U.K.

——— (2002): "Simulation Based Finite and Large Sample Tests in Multivariate Regressions," *Journal of Econometrics*, 111(2), 303–322.

DUFOUR, J.-M., L. KHALAF, AND M.-C. BEAULIEU (2003): "Exact Skewness-Kurtosis Tests for Multivariate Normality and Goodness-of-Fit in Multivariate Regressions with Application to Asset Pricing Models," *Oxford Bulletin of Economics and Statistics*, 65, 891–906.

DUFOUR, J.-M., L. KHALAF, J.-T. BERNARD, AND I. GENEST (2004): "Simulation-Based Finite-Sample Tests for Heteroskedasticity and ARCH Effects," *Journal of Econometrics*, 122(2), 317–347.

DUFOUR, J.-M., L. KHALAF, AND M. KICHIAN (2006a): "Inflation Dynamics and the New Keynesian Phillips Curve: An Identification Robust Econometric Analysis," *Journal of Economic Dynamics and Control*, 30(9-10), 1707–1727.

——— (2006b): "Structural Estimation and Evaluation of Calvo-Style Inflation Models," Discussion paper, Centre de recherche et développement en économique (CRDE), Université de Montréal, and Centre interuniversitaire de recherche en analyse des organisations (CIRANO), Montréal, Canada.

DUFOUR, J.-M., AND J. F. KIVIET (1996): "Exact Tests for Structural Change in First-Order Dynamic Models," *Journal of Econometrics*, 70, 39–68.

——— (1998): "Exact Inference Methods for First-Order Autoregressive Distributed Lag Models," *Econometrica*, 66, 79–104.

DUFOUR, J.-M., AND S. MAHSEREDJIAN (1993): "Tabulation of Farebrother's Test of Linear Restrictions: A Solution," *Econometric Theory*, 9, 697–702.

DUFOUR, J.-M., AND M. TAAMOUTI (2001): "Point-Optimal Instruments and Generalized Anderson-Rubin Procedures for Nonlinear Models," Discussion paper, C.R.D.E., Université de Montréal.

——— (2005): "Projection-Based Statistical Inference in Linear Structural Models with Possibly Weak Instruments," *Econometrica*, 73(4), 1351–1365.

——— (2006): "Further Results on Projection-Based Inference in IV Regressions with Weak, Collinear or Missing Instruments," *Journal of Econometrics*, forthcoming.

DWASS, M. (1957): "Modified Randomization Tests for Nonparametric Hypotheses," *Annals of Mathematical Statistics*, 28, 181–187.

GLESER, L. J., AND J. T. HWANG (1987): "The Nonexistence of $100(1-\alpha)$ Confidence Sets of Finite Expected Diameter in Errors in Variables and Related Models," *The Annals of Statistics*, 15, 1351–1362.

HALL, A. R., G. D. RUDEBUSCH, AND D. W.

WILCOX (1996): "Judging Instrument Relevance in Instrumental Variables Estimation," *International Economic Review*, 37, 283–298.

HILLIER, G. H. (1990): "On the Normalization of Structural Equations: Properties of Direction Estimators," *Econometrica*, 58, 1181–1194.

KIVIET, J., AND J.-M. DUFOUR (1997): "Exact Tests in Single Equation Autoregressive Distributed Lag Models," *Journal of Econometrics*, 80, 325–353.

KLEIBERGEN, F. (2001): "Testing Subsets of Structural Coefficients in the IV Regression Model," Discussion paper, Department of Quantitative Economics, University of Amsterdam.

——— (2002): "Pivotal Statistics for Testing Structural Parameters in Instrumental Variables Regression," *Econometrica*, 70(5), 1781–1803.

KOSCHAT, M. A. (1987): "A Characterization of the Fieller Solution," *The Annals of Statistics*, 15, 462–468.

MADDALA, G. S., AND J. JEONG (1992): "On the Exact Small Sample Distribution of the Instrumental Variable Estimator," *Econometrica*, 60, 181–183.

McManus, D. A., J. C. Nankervis, and N. E. Savin (1994): "Multiple Optima and Asymptotic Approximations in the Partial Adjustment Model," *Journal of Econometrics*, 62, 91–128.

Moreira, M. J. (2001): "Tests With Correct Size When Instruments Can Be Arbitrarily Weak," Discussion paper, Department of Economics, Harvard University, Cambridge, Massachusetts.

——— (2002): "A Conditional Likelihood Ratio Test for Structural Models," Discussion paper, Department of Economics, Harvard University, Cambridge, Massachusetts.

Nelson, C. R., and R. Startz (1990a): "The Distribution of the Instrumental Variable Estimator and its t-ratio When the Instrument is a Poor One," *Journal of Business*, 63, 125–140.

——— (1990b): "Some Further Results on the Exact Small Properties of the Instrumental Variable Estimator," *Econometrica*, 58, 967–976.

PERRON, B. (1999): "Semi-Parametric Weak Instrument Regressions with an Application to the Risk Return Trade-Off," Discussion Paper 0199, C.R.D.E., Université de Montréal.

PHILLIPS, P. C. B. (1984): "The Exact Distribution of LIML: I," *International Economic Review*, 25, 249–261.

——— (1985): "The Exact Distribution of LIML: II," *International Economic Review*, 26, 21–36.

——— (1989): "Partially Identified Econometric Models," *Econometric Theory*, 5, 181–240.

SARGAN, J. D. (1983): "Identification and Lack of Identification," *Econometrica*, 51, 1605–1633.

SHEA, J. (1997): "Instrument Relevance in Multivariate Linear Models: A Simple Measure," *Review of Economics and Statistics*, LXXIX, 348–352.

STAIGER, D., AND J. H. STOCK (1997): "Instrumental Variables Regression with Weak Instruments," *Econometrica*, 65(3), 557–586.

STARTZ, R., C. R. NELSON, AND E. ZIVOT (1999): "Improved Inference for the Instrumental Variable Estimator," Discussion paper, Department of Economics, University of Washington.

STOCK, J. H., AND J. H. WRIGHT (2000): "GMM with Weak Identification," *Econometrica*, 68, 1097–1126.

STOCK, J. H., J. H. WRIGHT, AND M. YOGO (2002): "A Survey of Weak Instruments and Weak Identification in Generalized Method of Moments," *Journal of Business and Economic Statistics*, 20(4), 518–529.

STOCK, J. H., AND M. YOGO (2002): "Testing for Weak Instruments in Linear IV Regression," Discussion Paper 284, N.B.E.R., Harvard University, Cambridge, Massachusetts.

WANG, J., AND E. ZIVOT (1998): "Inference on Structural Parameters in Instrumental Variables Regression with Weak Instruments," *Econometrica*, 66(6), 1389–1404.

ZIVOT, E., R. STARTZ, AND C. R. NELSON (1998):

"Valid Confidence Intervals and Inference in the Presence of Weak Instruments," *International Economic Review*, 39, 1119–1144.