Finite-sample inference, weak identification and macroeconometrics *

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1. Finite-sample inference in econometrics

General objective: to developing reliable tests and confidence sets (or intervals) in econometrics
in situations which involve test statistics:

1. complex finite-sample distributions;
2. nuisance parameters.

1.1. Monte Carlo test methods

Monte Carlo tests allow one to control perfectly the size of a test, even with a **small** number replications [Dwass (1957), Barnard (1963)].

- Dufour and Kiviet (1996, Journal of Econometrics)
- Kiviet and Dufour (1997a, Journal of Econometrics)
- Dufour and Kiviet (1998, Econometrica)
- Dufour, Farhat, Gardiol, and Khalaf (1998, Econometrics Journal)
- Dufour and Khalaf (2001, Baltagi, eds, Blackwell)

  Dufour and Khalaf (2002, Journal of Econometrics)
- Dufour, Khalaf, and Beaulieu (2003, Oxford Bulletin of Economics and Statistics)
- Dufour, Khalaf, Bernard, and Genest (2004, Journal of Econometrics)
- Dufour (2006, Journal of Econometrics): generalized theory of MC tests

- Beaulieu, Dufour, and Khalaf (2006, Journal of Business and Economic Statistics, forth.)

1.2. Nuisance parameter elimination

1. Transformations
2. Conditioning

- Campbell and Dufour (1991, Economics Letters)
- Campbell and Dufour (1995, Review of Economics and Statistics)
- Dufour and Kiviet (1996, Journal of Econometrics)
- Kiviet and Dufour (1997b, Journal of Econometrics)
- Dufour and Kiviet (1998, Econometrica)
- Campbell and Dufour (1997, International Economic Review)
- Dufour and Torrès (1998, Dekker)
- Dufour and Torrès (2000, Journal of Econometrics)
- Dufour, Farhat, and Hallin (2003, Journal of Econometrics)

1.3. Bound procedures

1.3.1. Bounding the statistic of interest by other statistics

- Dufour (1989, Econometrica)
- Dufour (1990, Econometrica)
- Dufour and Khalaf (2002, Journal of Econometrics)
- Dufour (2006, Journal of Econometrics): Maximized Monte Carlo (MMC) tests
- Dufour and Jouini (2006, Journal of Econometrics): MMC tests applied to VAR models

1.3.2. Bounding tail areas by some function

Exponential inequalities, Chebyshev inequalities, Berry-Esséen bounds

- Dufour (1991, Hackl-Westlund, eds., Springer)
- Dufour and Mahseredjian (1993, Econometric Theory)
- Dufour and Hallin (1991, Econometric Theory)
- Dufour and Hallin (1992a, Econometric Theory)
- Dufour and Hallin (1992b, Journal of Statistical Planning and Inference)
- Dufour and Hallin (1993, JASA)

1.3.3. Projection techniques

- Dufour (1990, Econometrica)
- Dufour (1997, Econometrica)
- Abdelkhalek and Dufour (1998, Review of Economics and Statistics)
- Dufour and Kiviet (1998, Econometrica)
- Dufour and Jasiak (2001, International Economic Review)
- Dufour and Khalaf (2002)
- Dufour and Taamouti (2005b, Econometrica)
- Dufour and Taamouti (2006, Journal of Econometrics, forth.)

1.3.4. Sequential confidence procedure

- Dufour (1990, Ecnometrica)
- Dufour, Hallin, and Mizera (1998, Journal of Non-parametric Statistics)
- Dufour and Kiviet (1998, Econometrica)
- Beaulieu, Dufour, and Khalaf (2006, Journal of Business and Economic Statistics, forth.)

2. Weak identification

Identification failure: situation where several values of model parameters correspond to the same data DGP.
Weak identification: situation where we are close to identification failure.

Several authors in the pas have noted that usual asymptotic approximations are not valid or lead to very inaccurate results when parameters of interest are close to regions where these parameters are not anymore identifiable

Surveys:

- Stock, Wright, and Yogo (2002, Journal of Business and Economic Statistics)
- Dufour (2003, Canadian Journal of Economics)

1. Theoretical results show that the distributions of various estimators depend in a complicated way upon unknown nuisance parameters. So they are difficult to interpret.
2. When identification conditions do not hold, standard asymptotic theory for estimators and test statistics typically collapses.
3. With weak instruments,
   (a) 2SLS becomes heavily biased (in the same direction as OLS),
   (b) distribution of 2SLS is quite far the normal distribution (e.g., bimodal).
4. Standard Wald-type procedures based on asymptotic standard errors become fundamentally unreliable or very unreliable in finite samples.
5. Problems were strikingly illustrated by the reconsideration by Bound, Jaeger, and Baker (1995, Journal of the American Statistical Association) of a study on returns to education by Angrist and Krueger (1991, QJE):

   329000 observations;

replacing the instruments used by Angrist and Krueger (1991, QJE) with randomly generated instruments (totally irrelevant) produced very similar point estimates and standard errors;

indicates that the instruments originally used were weak.

Crucial to use finite-sample approaches to produce reliable inference.

Finite-sample approaches to inference on models involving weak identification

- Dufour (1997, Econometrica)
- Dufour and Jasiak (2001, International Economic Review)
- Dufour and Taamouti (2005b, Econometrica)
- Dufour and Taamouti (2006, Journal of Econometrics, forth.)

1. Procedures robust to lack of identification (or weak identification)
2. Limited information methods which do not require a complete formulation of the model [robustness to missing instruments]

Applications

1. Education and labour economics

   (a) Students' achievements and self-esteem
   [Dufour and Jasiak (2001, International Economic Review)]

   (b) Education and earnings
   [Dufour and Taamouti (2006, Journal of Econometrics, forth.)]

2. Financial econometrics

   (a) Black's CAPM
   [Beaulieu, Dufour, and Khalaf (2005)]

   (b) Stochastic volatilty models
   [Dufour and Valéry (2005)]

3. Macroeconomics

   (a) Tobin's q
   [Dufour and Jasiak (2001, International Economic Review)]

   (b) Trade and growth
   [Dufour and Taamouti (2006, Journal of Econometrics, forth.)]

(c) New Keynesian Phillips curves
[Dufour, Khalaf, and Kichian (2006a, Journal of Economic Dynamics and Control), Dufour, Khalaf, and Kichian (2006b)]

3. Macroeconometrics

3.1. Distribution-free inference in models with regressors involving feedback

- Campbell and Dufour (1991, Economics Letters)
- Campbell and Dufour (1995, Review of Economics and Statistics)
- Campbell and Dufour (1997, International Economic Review)
- Coudin and Dufour (2005a)
- Coudin and Dufour (2005b)

3.2. Causality analysis, VAR and VARMA models

- Boudjellaba, Dufour, and Roy (1992, JASA)
- Dufour and Tessier (1993)
- Boudjellaba, Dufour, and Roy (1994, Journal of Econometrics)
- Dufour and Tessier (1997)
- Dufour and Renault (1998, Econometrica): causality at various horizons
- Dufour and Jouini (2006, Journal of Econometrics): exact inference for large VAR models (bootstrap)
- Dufour, Pelletier, and Renault (2006, Journal of Econometrics): tests for causality at various horizons
- Dufour and Pelletier (2005): VARMA identification and modelling
- Dufour and Taamouti (2005a): causality measures (bootstrap)

- Dufour, Garcia, and Taamouti (2006)

3.3. New Keynesian Phillips curves

- Dufour, Khalaf, and Kichian (2006a, Journal of Economic Dynamics and Control): Gali-Gertler model
- Dufour, Khalaf, and Kichian (2006b): Calvo
- Dufour, Khalaf, and Kichian (2006c): Lindé model

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