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ECONOMETRIC THEORY
EXERCISES 1
DECISION THEORY

Reference: Gouriéroux and Monfort (1995, Chapter 2)

1. Describe the main statistical problems as decision problems.
   (a) Explain the difference between a *nonrandomized* decision rule and a *randomized* decision rule.
   (b) Define the risk function for each one of these two types of rule.
   (c) When is a decision rule *admissible*?
2. If a randomized decision rule m_1 is preferable to another randomized decision rule m_2, show that m_1 is preferable to m_2 in the Bayesian sense.
3. Is a decision rule optimal in the Bayesian sense always admissible? Justify your answer. [You can limit yourself to the case of a discrete distribution.]
4. Exercise 2.1 in Gouriéroux and Monfort (1995, chap. 2).
5. Exercise 2.2 in Gouriéroux and Monfort (1995, chap. 2).
6. Exercise 2.3 in Gouriéroux and Monfort (1995, chap. 2).
7. Let Y be a Bernoulli random variable $B(1, \theta)$ where θ can take the values $\frac{1}{3}$ or $\frac{1}{2}$. We consider the problem of estimating θ using a single observation Y.
   (a) How many nonrandomized decision rules do exist for this problem? Describe these rules.
   (b) Describe the set of randomized rules for this problem.

(c) Compute the risk function associated with each one of the nonrandomized decision rules. Represent in a graph the risks associated with the different rules. Which rules are admissible ? [Remark: there is an error in Figure 2.1 of Gouriéroux and Monfort (1995, page 57).]

(d) Compute the risk function associated with each one of the randomized decision rules. Represent in a graph these different risks. Which rules are admissible ?

References

GOURIÉROUX, C., AND A. MONFORT (1995): *Statistics and Econometric Models, Volumes One and Two*. Cambridge University Press, Cambridge, U.K.