# Factor-Augmented VARMA Models with Macroeconomic Applications*

Jean-Marie Dufour †
McGill University

Dalibor Stevanović ‡
Université du Québec à Montréal

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† William Dow Professor of Economics, McGill University, Centre interuniversitaire de recherche en analyse des organisations (CIRANO), and Centre interuniversitaire de recherche en économie quantitative (CIREQ). Mailing address: Department of Economics, McGill University, Leacock Building, Room 519, 855 Sherbrooke Street West, Montréal, Québec H3A 2T7, Canada. TEL: (1) 514 398 4400 ext. 09156; FAX: (1) 514 398 4800; e-mail: jean-marie.dufour@mcgill.ca . Web page: http://www.jeanmariedufour.com

‡ Département des sciences économiques, Université du Québec à Montréal. Mailing address: Université du Québec à Montréal, Département des sciences économiques, 315, rue Ste-Catherine Est, Montréal, QC, H2X 3X2. e-mail: dstevanovic.econ@gmail.com

## ABSTRACT

We study the relationship between VARMA and factor representations of a vector stochastic process. We observe that, in general, vector time series and factors cannot both follow finite-order VAR models. Instead, a VAR factor dynamics induces a VARMA process, while a VAR process entails VARMA factors. We propose to combine factor and VARMA modeling by using factor-augmented VARMA (FAVARMA) models. This approach is applied to forecasting key macroeconomic aggregates using large U.S. and Canadian monthly panels. The results show that FAVARMA models yield substantial improvements over standard factor models, including precise representations of the effect and transmission of monetary policy.



**Key words**: factor analysis, VARMA process, forecasting, structural analysis.
**Journal of Economic Literature classification:** C32, C51, C52, C53.

SUMMARY

We study the relationship between VARMA and factor representations of a vector stochastic process, and we propose to use factor-augmented VARMA (FAVARMA) models as an alternative to usual VAR models. We start by observing that vector time series and the associated factors do not both follow a finite-order VAR process, except in very special cases. When factors are defined as linear combinations of observable series, the observable series follows a VARMA process, not a finite-order VAR as typically assumed. Second, even if the factors follow a finite-order VAR model, this entails a VARMA representation for the observable series. In view of these observations, we propose to use a FAVARMA framework which combines two dimension reduction techniques in order to represent the dynamic interactions between a large number of time series: factor analysis and VARMA modeling. We apply this approach in two out-of-sample forecasting exercises using large U.S. and Canadian monthly panels. The results show that VARMA factors provide better forecasts for several key macroeconomic aggregates relative to standard factor models. Finally, we estimate the effect of monetary policy using the data and the identification scheme of Bernanke, Boivin and Eliasz (2005). We find that impulse responses from a parsimonious 6-factor FAVARMA(2,1) model give an accurate and plausible picture of the effect and transmission of monetary policy in the U.S. To get similar responses from a standard FAVAR model, the Akaike information criterion leads to a lag order of 14. The FAVARMA model requires the estimation of 84 coefficients in order to represent the system dynamics, while the corresponding FAVAR model includes 510 VAR parameters.

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# 1. INTRODUCTION

As information technology improves, the availability of economic and financial time series grows in terms of both time and cross-section size. However, a large amount of information can lead to a dimensionality problem when standard time series tools are used. Since most of these series are correlated, at least within some categories, their co-variability and information content can be approximated by a smaller number of variables. A popular way to address this issue is to use "large dimensional approximate factor analysis", an extension of classical factor analysis which allows for limited cross-section and time correlations among idiosyncratic components.

While factor models were introduced in macroeconomics and finance by Sargent and Sims (1977), Geweke (1977), and Chamberlain and Rothschild (1983), the literature on the large factor models starts with Forni, Hallin, Lippi and Reichlin (2000) and Stock and Watson (2002). Further theoretical advances were made, among others, by Bai and Ng (2002), Bai (2003), and Forni, Hallin, Lippi and Reichlin (2004). These models can be used to forecast macroeconomic aggregates [Stock and Watson (2002b), Forni, Hallin, Lippi and Reichlin (2005), Banerjee, Marcellino and Masten (2006)], structural macroeconomic analysis [Bernanke et al. (2005), Favero, Marcellino and Neglia (2005)], for nowcasting and economic monitoring [Giannone, Reichlin and Small (2008), Aruoba, Diebold and Scotti (2009)], to deal with weak instruments [Bai and Ng (2010), Kapetanios and Marcellino (2010)], and the estimation of dynamic stochastic general equilibrium models [Boivin and Giannoni (2006)].

Vector autoregressive moving-average (VARMA) models provide another way to obtain a parsimonious representation of a vector stochastic process. VARMA models are especially appropriate in forecasting, since they can represent the dynamic relations between time series while keeping the number of parameters low; see Lütkepohl (1987) and Boudjellaba, Dufour and Roy (1992). Further, VARMA structures emerge as reduced-form representations of structural models in macroeconomics. For instance, the linear solution of a standard dynamic stochastic general equilibrium model generally implies a VARMA representation on the observable endogenous variables [Ravenna (2006), Komunjer and Ng (2011), and Poskitt (2011)].

In this paper, we study the relationship between VARMA and factor representations of a vector stochastic process, and we propose a new class of factor-augmented VARMA models. We start by observing that, in general, multivariate time series and the associated factors do not typically both follow finite-order VAR processes. When the factors are obtained as linear combinations of observable series, the dynamic process obeys a VARMA model, not a finite-order VAR as usually assumed in the literature. Further, if the latent factors follow a finite-order VAR process, this implies a VARMA representation for the observable series. Consequently, we propose to combine two techniques for representing in a parsimonious way the dynamic interactions between a huge number of time series: dynamic factor reduction and VARMA modelling. Thus lead us to consider factor-augmented VARMA (FAVARMA) models.Besides parsimony, the class of VARMA models is closed under marginalization and linear transformations (in contrast with VAR processes). This represents an additional advantage if the number of factors is underestimated.

The importance of the factor process specification depends on the technique used to estimate the factor model and the research goal. In the two-step method developed by Stock and Watson (2002), the factor process does not matter for the approximation of factors, but this might be an issue if we use a likelihood-based technique which relies on a completely specified process. Moreover, if predicting observable variables depends on factor forecasting, a reliable and parsimonious approximation of the factor dynamic process is important. In Deistler, Anderson, Filler, Zinner and Chen (2010), the authors study identification of the generalized dynamic factor model where the common component has a singular rational spectral density. Under the assumption that transfer functions are tall and zeroless (*i.e.,* the number of common shocks is less than the number of static factors), they argue that static factors have a finite-order AR singular representation which can be estimated by generalized Yule-Walker equations. Note that Yule-Walker equations are not unique for such systems, but Deistler, Filler and Funovits (2011) propose a particular canonical form for estimation purposes.

After showing that FAVARMA models yield a theoretically consistent specification, we study whether VARMA factors can help in forecasting time series. We compare the forecasting performance (in terms of MSE) of four FAVARMA specifications, with standard AR$(p)$, ARMA$(p,q)$ and factor models where the factor dynamics is approximated by a finite-order VAR. An out-of-sample forecasting exercise is performed using a U.S. monthly panel from Boivin, Giannoni and Stevanović (2009).

The results show that VARMA factors help in predicting several key macroeconomic aggregates, relative to standard factor models, and across different forecasting horizons. We find important gains, up to a reduction of 42% in MSE, when forecasting the growth rates of industrial production, employment and consumer price index inflation. In particular, the FAVARMA specifications generally outperform the VAR-factor forecasting models. We also report simulation results which show that VARMA factor modelling noticeably improves forecasting in finite samples.

Finally, we perform a structural factor analysis exercise. We estimate the effect of a monetary policy shock using the data and identification scheme of Bernanke et al. (2005). We find that impulse responses from a parsimonious 6-factor FAVARMA$(2,1)$ model give a precise and plausible picture of the effect and transmission of monetary policy in the U.S. To get similar responses from a standard FAVAR model, the Akaike information criterion leads to a lag order of 14. So we need to estimate 84 coefficients governing the factor dynamics in the FAVARMA framework, while the FAVAR model requires 510 VAR parameters.

In Section 2, we summarize some important results on linear transformations of vector stochastic processes and present four identified VARMA forms. In Section 3, we study the link between VARMA and factor representations. The FAVARMA model is proposed in Section 4, and estimation is discussed in Section 5. Monte Carlo simulations are discussed in Section 6. The empirical forecasting exercise is presented in Section 7, and the structural analysis in Section 8. Proofs and simulation results are reported in Appendix.

## 2. FRAMEWORK

In this section, we summarize a number of important results on linear transformations of vector stochastic processes, and we present four identified VARMA forms we will use in forecasting applications.

## 2.1. Linear transformations of vector stochastic processes

Exploring the features of transformed processes is important since data are often obtained by temporal and spatial aggregation, and/or transformed through linear filtering techniques, before they are used to estimate models and evaluate theories. In macroeconomics, researchers model dynamic interactions by specifying a multivariate stochastic process on a small number of economic indicators. Hence, they work on marginalized processes, which can be seen as linear transformations of the original series. Finally, dimension-reduction methods, such as principal components, lead one to consider linear transformations of the observed series. Early contributions on these issues include Zellner and Palm (1974), Rose (1977), Wei (1978), Abraham (1982), and Lütkepohl (1984).

The central result we shall use focuses on linear transformations of a $N$-dimensional, stationary, strictly indeterministic stochastic process. Suppose $X_t$ satisfies the model

$$X_t = \sum_{j=0}^{\infty} \Psi_j \varepsilon_{t-j} = \Psi(L)\varepsilon_t\,, \quad \Psi_0 = I_K\,, \tag{2.1}$$

where $\varepsilon_t$ is a weak white noise, with $E(\varepsilon_t) = 0$, $E(\varepsilon_t \varepsilon_t') = \Sigma_\varepsilon$, $\det[\Sigma_\varepsilon] > 0$, $E(X_t X_t') = \Sigma_X$, $E(X_t X_{t+h}') = \Gamma_X(h)$, $\Psi(L) = \sum_{i=0}^{\infty} \Psi_i L^i$ and $\det[\Psi(z)] \neq 0$ for $|z| < 1$. (2.1) can be interpreted as the Wold representation of $X_t$, in which case $\varepsilon_t = X_t - P_L[X_t \,|\, X_{t-1}, X_{t-2}, \ldots]$ and $P_L[X_t \,|\, X_{t-1}, X_{t-2}, \ldots]$ is the best linear forecast of $X_t$ based on its own past (*i.e.*, $\varepsilon_t$ is the *innovation process* of $X_t$). Consider the following linear transformation of $X_t$:

$$F_t = CX_t \tag{2.2}$$

where $C$ is a $K \times N$ matrix of rank $K$. Then $F_t$ is also stationary, indeterministic and has zero mean, so it has an MA representation of the form:

$$F_t = \sum_{j=0}^{\infty} \Phi_j v_{t-j} = \Phi(L) v_t\,, \quad \Phi_0 = I_K\,, \tag{2.3}$$

where $v_t$ is $K$-dimensional white noise with $E(v_t v_t') = \Sigma_v$. These properties hold whenever $X_t$ is a vector stochastic process with an MA representation. If it is invertible, finite and infinite-order VAR processes are covered.

In practice, only a finite number of parameters can be estimated. Consider the MA($q$) process

$$X_t = \varepsilon_t + M_1 \varepsilon_{t-1} + \cdots + M_q \varepsilon_{t-q} = M(L)\varepsilon_t \tag{2.4}$$

with $\det[M(z)] \neq 0$ for $|z| < 1$ and nonsingular white noise noise covariance matrix $\Sigma_\varepsilon$, and a $K \times N$ matrix $C$ with rank $K$. Then, the transformed process $F_t = CX_t$ has an invertible MA($q_*$) representation

$$F_t = v_t + N_1 v_{t-1} + \cdots + N_{q_*} v_{t-q_*} = N(L) v_t \tag{2.5}$$

with $\det[N(z)] \neq 0$ for $|z| < 1$, where $v_t$ is a $K$-dimensional white noise with nonsingular matrix $\Sigma_v$, each $N_i$ is a $K \times K$ coefficient matrix, and $q_* \leq q$.

Some conditions in the previous results can be relaxed. The nonsingularity of the covariance matrix $\Sigma_\varepsilon$ and the full rank of $C$ are not necessary so there may be exact linear dependencies among the components of $X_t$ and $F_t$ [see Lütkepohl (1984b)]. It is also possible that $q_* < q$.

It is well known that weak VARMA models are closed under linear transformations. Let $X_t$ be an $N$-dimensional, stable, invertible VARMA$(p, q)$ process

$$\Phi(L)X_t = \Theta(L)\varepsilon_t \tag{2.6}$$

and let C be a $K \times N$ matrix of rank $K < N$. Then $F_t = CX_t$ has a VARMA$(p_*, q_*)$ representation with $p_* \leq (N-K+1)p$ and $q_* \leq (N-K)p+q$; see Lütkepohl (2005, Corollary 11.1.2). A linear transformation of a finite-order VARMA process still has a finite-order VARMA representation, but with possibly higher autoregressive and moving-average orders.

When modeling economic time series, the most common specification is a finite-order VAR. Therefore, it is important to notice that this class of models is not closed with respect to linear transformations reducing the dimensions of the original process.

## 2.2. Identified VARMA models

An identification problem arises since the VARMA representation of $X_t$ is not unique. There are several ways to identify the process in (2.6). In the following, we state four unique VARMA representations: the well-known final-equation form and three representations proposed in Dufour and Pelletier (2013).

**Definition 2.1** FINAL AR EQUATION FORM (FAR). *The VARMA representation in* (2.6) *is said to be in final AR equation form if* $\Phi(L) = \phi(L)I_N$, *where* $\phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$ *is a scalar polynomial with* $\phi_p \neq 0$.

**Definition 2.2** FINAL MA EQUATION FORM (FMA). *The VARMA representation in* (2.6) *is said to be in final MA equation form if* $\Theta(L) = \theta(L)I_N$, *where* $\theta(L) = 1 - \theta_1 L - \cdots - \theta_q L^q$ *is a scalar polynomial with* $\theta_q \neq 0$.

**Definition 2.3** DIAGONAL MA EQUATION FORM (DMA). *The VARMA representation in* (2.6) *is said to be in diagonal MA equation form if* $\Theta(L) = \mathrm{diag}[\theta_{ii}(L)] = I_N - \Theta_1 L - \cdots - \Theta_q L^q$, *where* $\theta_{ii}(L) = 1 - \theta_{ii,1}L - \cdots - \theta_{ii,q_i}L^{q_i}$, $\theta_{ii,q_i} \neq 0$, *and* $q = \max_{1\leq i\leq N}(q_i)$.

**Definition 2.4** DIAGONAL AR EQUATION FORM (DAR). *The VARMA representation in* (2.6) *is said to be in diagonal AR equation form if* $\Phi(L) = \mathrm{diag}[\phi_{ii}(L)] = I_N - \Phi_1 L - \cdots - \Phi_p L^p$, *where* $\phi_{ii}(L) = 1 - \phi_{ii,1}L - \cdots - \phi_{ii,p_i}L^{p_i}$, $\phi_{ii,p_i} \neq 0$, *and* $p = \max_{1\leq i\leq N}(p_i)$.

The identification of these VARMA representations is discussed in Dufour and Pelletier (2013, Section 3). In particular, the identification of diagonal MA form is established under the simple assumption of no common root.

From standard results on the linear aggregation of VARMA processes [see, e.g., Zellner and Palm (1974), Rose (1977), Wei (1978), Abraham (1982), and Lütkepohl (1984)], it is easy to see that an aggregated process such as $F_t$ also has an identified VARMA representation in final AR or MA equation form. But this type of representation may not be attractive for several reasons. First, it is far from the usual VAR model, because it excludes lagged values of other variables in each equation. Moreover, the AR coefficients are the same in all equations, which typically leads to a high-order AR polynomial. Second, the interaction between different variables is modeled through the MA part of the model, and may be difficult to assess in empirical and structural analysis.

The diagonal MA form is especially appealing. In contrast with the echelon form [Deistler and Hannan (1981), Hannan and Deistler (1988), and Lütkepohl (1991, Chapter 7)], it is relatively simple and intuitive. In particular, there is no complex structure of zero off-diagonal elements in the AR and MA operators. For practitioners, this is quite appealing since adding lags of $\varepsilon_{it}$ to the $i^{th}$ equation is a simple natural extension of the VAR model. The MA operator has a simple diagonal form, so model nonlinearity is reduced and estimation becomes numerically simpler.

# 3. VARMA AND FACTOR REPRESENTATIONS

In this section, we study the link between VARMA and factor representations of a vector stochastic process $X_t$, and the dynamic process of the factors. In the theorems below, we suppose that $X_t$ is a $N$-dimensional regular (strictly indeterministic) discrete-time process in $\mathbb{R}^N : X = \{X_t : t \in \mathbb{R}^N,\ t \in \mathbb{Z}\}$ with Wold representation (2.1). In Theorem **3.1**, we postulate a factor model for $X_t$ where factors follow a finite-order VAR process:

$$X_t = \Lambda F_t + u_t \tag{3.1}$$

where $\Lambda$ is an $N \times K$ matrix of factor loadings with rank $K$, and $u_t$ is a (weak) white noise process with covariance matrix $\Sigma_u$ such that

$$\mathsf{E}[F_t u_t'] = 0 \text{ for all } t. \tag{3.2}$$

We now show that finite-order VAR factors induce a finite-order VARMA process for the observable series. Proofs are supplied in the Appendix.

**Theorem 3.1** OBSERVABLE PROCESS INDUCED BY FINITE-ORDER VAR FACTORS. *Suppose $X_t$ satisfies the assumptions* (3.1) - (3.2) *and $F_t$ follows the VAR$(p)$ process*

$$F_t = \Phi(L) F_{t-1} + a_t \tag{3.3}$$

*such that $e_t = [u_t \vdots a_t]'$ is a (weak) white noise process with*

$$\mathsf{E}[F_{t-j} e_t'] = 0 \, for \ j \geq 1,\ \forall t, \tag{3.4}$$

*$\Phi(L) = \Phi_1 L - \cdots - \Phi_p L^p$, and the equation* $\det[I_K - \Phi(z)] = 0$ *has all its roots outside the unit circle. Then, for all $t$, $\mathsf{E}[X_{t-j} e_t'] = 0$ for $j \geq 1$, and $X_t$ has the following representations:*

$$A(L) X_t = B(L) e_t\,, \tag{3.5}$$

$$A(L)X_t = \bar{\Psi}(L)\varepsilon_t\,, \tag{3.6}$$

*where* $A(L) = [I - \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' L]$, $B(L) = [A(L) \vdots \Lambda]$, $\bar{\Psi}(L) = \sum_{j=0}^{p+1} \bar{\Psi}_j L^j$ *with* $\bar{\Psi}_j = \sum_{i=0}^{p+1} A_i \Psi_{j-i}$, *the matrices* $\Psi_j$ *are the coefficients of the Wold representation* (2.1), *and* $\varepsilon_t$ *is the innovation process of* $X_t$.

This result can be extended to the case where the factors have VARMA representations. It is not surprising that the induced process for $X_t$ is again a finite-order VARMA, though possibly with a different MA order. This is summarized in the following theorem.

**Theorem 3.2** OBSERVABLE PROCESS INDUCED BY VARMA FACTORS. *Suppose* $X_t$ *satisfies the assumptions* (3.1) - (3.2) *and* $F_t$ *follows the VARMA*$(p, q)$ *process*

$$F_t = \Phi(L)F_{t-1} + \Theta(L)a_t \tag{3.7}$$

*where* $e_t = [u_t \vdots a_t]'$ *is a* (*weak*) *white noise process which satisfies the orthogonality condition* (3.4), $\Phi(L) = \Phi_1 L - \cdots - \Phi_p L^p$, $\Theta(L) = I_K - \Theta_1 L - \cdots - \Theta_q L^q$, *and the equation* $\det[I_K - \Phi(z)] = 0$ *has all its roots outside the unit circle. Then* $X_t$ *has representations of the form* (3.5) *and* (3.6), *with* $B(L) = [A(L) \vdots \Lambda\Theta(L)]$, $\bar{\Psi}(L) = \sum_{j=0}^{p_*} \bar{\Psi}_j L^j$, $\bar{\Psi}_j = \sum_{i=0}^{p_*} A_i \Psi_{j-i}$, *and* $p_* = \max(p+1, q)$.

Note that the usual invertibility assumption on the factor VARMA process (3.7) is not required. The next issue we consider concerns the factor representation of $X_t$. What are the implications of the underlying structure of $X_t$ on the representation of latent factors when the latter are calculated as linear transformations of $X_t$? This is summarized in the following theorem.

**Theorem 3.3** DYNAMIC FACTOR MODELS ASSOCIATED WITH VARMA PROCESSES. *Suppose* $F_t = CX_t$, *where* $C$ *is a* $K \times N$ *full row rank matrix. Then the following properties hold:*

(*i*) *if* $X_t$ *has a VARMA*$(p, q)$ *representation as in* (2.6), *then* $F_t$ *has VARMA*$(p_*, q_*)$ *representation with* $p_* \leq (N - K + 1)p$ *and* $q_* \leq q + (N - K)p$;

(*ii*) *if* $X_t$ *has a VAR(p) representation, then* $F_t$ *has VARMA*$(p_*, q_*)$ *representation with* $p_* \leq Np$ *and* $q_* \leq (N-1)p$;

(*iii*) *if* $X_t$ *has an MA representation as in* (2.4), *then* $F_t$ *has an MA*$(q_*)$ *representation with* $q \leq q_*$.

From the Wold decomposition of common components, Deistler et al. (2010) argue that latent variables can have ARMA or state-space representations, but given the singularity and zero-free nature of transfer functions, they can also be modeled as finite-order singular AR processes. Theorem **3.3** does not assume the existence of a dynamic factor structure, so it holds for any linear aggregation of $X_t$.

Arguments in favor of using a FAVARMA specification can be summarized as follows.

(i) Whenever $X_t$ follows a VAR or a VARMA process, the factors defined through a linear cross-sectional transformation (such as principal components) follow a VARMA process. Moreover, a VAR or VARMA-factor structure on $X_t$ entails a VARMA structure for $X_t$.

(ii) VARMA representations are more parsimonious, so they easily lead to more efficient estimation and tests. As shown in Dufour and Pelletier (2013), the introduction of the MA operator allows for a reduction of the required AR order so we can get more precise estimates. Moreover, in terms of forecasting accuracy, VARMA models have theoretical advantages over the VAR representation [see Lütkepohl (1987)].

(iii) The use of VARMA factors can be viewed from two different perspectives. First, if we use factor analysis as a dimension-reduction method, a VARMA specification is a natural process for factors (Theorem **3.3**). Second, if factors are given a deep ("structural") interpretation, the factor process has intrinsic interest, and a VARMA specification on factors – rather than a finite-order VAR – is an interesting generalization motivated by usual arguments of theoretical coherence, parsimony, and marginalization. In particular, even if $F_t$ has a finite-order VAR representation, subvectors of $F_t$ typically follow a VARMA process.

# 4. FACTOR-AUGMENTED VARMA MODELS

We have shown that the observable VARMA process generally induces a VARMA representation for factors, not a finite-order VAR. Following these results, we propose to consider factor-augmented VARMA (FAVARMA) models. Following the notation of Stock and Watson (2005), the dynamic factor model (DFM) where factors have a finite-order VARMA$(p_f, q_f)$ representation can be written as

$$\begin{aligned} X_{it} &= \tilde{\lambda}_i(L) f_t + u_{it}\,, && (4.1)\\ u_{it} &= \delta_i(L) u_{i,t-1} + \nu_{it}\,, && (4.2)\\ f_t &= \Gamma(L) f_{t-1} + \Theta(L)\eta_t\,, \quad i = 1,\ldots,N, \quad t = 1,\ldots,T, && (4.3) \end{aligned}$$

where $f_t$ is $q \times 1$ factor vector, $\tilde{\lambda}_i(L)$ is a $1 \times q$ vector of lag polynomials, $\tilde{\lambda}_i(L) = (\tilde{\lambda}_{i1}(L), \ldots, \tilde{\lambda}_{iq}(L))$, $\tilde{\lambda}_{ij}(L) = \sum_{k=0}^{p_{i,j}} \tilde{\lambda}_{i,j,k} L^k$, $\delta_i(L)$ is a $p_{x,i}$-degree lag polynomial, $\Gamma(L) = \Gamma_1 L + \cdots + \Gamma_{p_f} L^{p_f}$, $\Theta(L) = I - \Theta_1 L - \cdots - \Theta_{q_f} L^{q_f}$, and $\nu_{it}$ is a $N$-dimensional white noise uncorrelated with the $q$-dimensional white noise process $\eta_t$. The exact DFM is obtained if the following assumption is satisfied:

$$\mathsf{E}(u_{it} u_{js}) = 0\,,\ \forall i, j, t, s, \quad i \neq j.$$

We obtain the approximate DFM by allowing for cross-section correlations among the idiosyncratic components as in Stock and Watson (2005). We assume the idiosyncratic errors $\nu_{it}$ are uncorrelated with the factors $f_t$ at all leads and lags.

On premultiplying both sides of (4.1) by $1 - \delta_i(L)$, we get the DFM with serially uncorrelated idiosyncratic errors:

$$X_{it} = \lambda_i(L) f_t + \delta_i(L) X_{it-1} + \nu_{it} \qquad (4.4)$$

where $\lambda_i(L) = [1 - \delta_i(L)L]\tilde{\lambda}_i(L)$. Then, we can rewrite the DFM in the following form:

$$
\begin{aligned}
X_t &= \lambda(L) f_t + D(L) X_{t-1} + \nu_t \,, && (4.5)\\
f_t &= \Gamma(L) f_{t-1} + \Theta(L)\eta_t \,, && (4.6)
\end{aligned}
$$

where

$$
\lambda(L) = \begin{bmatrix} \lambda_1(L) \\ \vdots \\ \lambda_n(L) \end{bmatrix}, \quad D(L) = \begin{bmatrix} \delta_1(L) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \delta_n(L) \end{bmatrix}, \quad \nu_t = \begin{bmatrix} \nu_{1t} \\ \vdots \\ \nu_{nt} \end{bmatrix}.
$$

To obtain the static version, we suppose that $\lambda(L)$ has degree $p-1$, and let $F_t = [f_t', f_{t-1}', \dots, f_{t-p+1}']'$, where the dimension of $F_t$ is $K$, with $q \le K \le qp$. Then,

$$
\begin{aligned}
X_t &= \Lambda F_t + u_t, && (4.7)\\
u_t &= D(L) u_{t-1} + \nu_t, && (4.8)\\
F_t &= \Phi(L) F_{t-1} + G\Theta(L)\eta_t, && (4.9)
\end{aligned}
$$

where $\Lambda$ is a $N \times K$ matrix where the $i$-th row consists of coefficients of $\tilde{\lambda}_i(L)$, $\Phi(L)$ contains coefficients of $\Gamma(L)$ and zeros, and $G$ is a $K \times q$ matrix which loads (structural) shocks $\eta_t$ to static factors (it consists of 1's and 0's). Note that if $\Theta(L) = I$ we obtain the static factor model which has been used to forecast time series [Stock and Watson (2002b), Stock and Watson (2006), Boivin and Ng (2005)] and study the impact of monetary policy shocks in a FAVAR model [Bernanke et al. (2005), Boivin et al. (2009)].

# 5. ESTIMATION

Several estimation methods have been proposed for factor models and VARMA processes (separately). One possibility is to estimate the system (4.7)-(4.9) simultaneously after making distributional assumptions on the error terms. This method is already computationally difficult when the factors have a simple VAR structure. Adding the MA part to the factor process makes this task even more difficult, for estimating VARMA models is typically not easy.

We use here the two-step Principal Component Analysis (PCA) estimation method; see Stock and Watson (2002) and Bai and Ng (2008) for theoretical results concerning the PCA estimator. In the first step, $\hat{F}_t$ are computed as $K$ principal components of $X_t$. In the second step, we estimate the VARMA representation (4.9) on $\hat{F}_t$. The number of factors can be estimated through different procedures proposed by Amengual and Watson (2007), Bai and Ng (2002), Bai and Ng (2007), Hallin and Liska (2007), and Onatski (2009). In forecasting we estimate the number of factors using the Bayesian information criterion as in Stock and Watson (2002b), while the number of factors in the structural FAVARMA model is the same as in Bernanke et al. (2005).

The standard estimation methods for VARMA models are maximum likelihood and nonlinear least squares. Unfortunately, these methods require nonlinear optimization, which may not be feasible when the number of parameters is large. Here, we use the GLS method proposed in Dufour and Pelletier (2013), which generalizes the regression-based estimation method introduced by Hannan and Rissanen (1982). Consider a $K$-dimensional zero mean process $Y_t$ generated by the VARMA$(p,q)$ model:

$$A(L)Y_t = B(L)U_t \tag{5.1}$$

where $A(L) = I_K - A_1L - \cdots - A_pL^p$, $B(L) = I_K - B_1L - \cdots - B_qL^q$, and $U_t$ is a weak white noise. Assume $\det[A(z)] \neq 0$ for $|z| \leq 1$ and $\det[B(z)] \neq 0$ for $|z| \leq 1$ so the process $Y_t$ is stable and invertible. Set $A_k = [a'_{1\bullet,k}, \ldots, a'_{K\bullet,k}]'$, $k = 1, \ldots, K$, where $a_{j\bullet,k}$ is the $j$-th row of $A_k$, and $B(L) = \mathrm{diag}[b_{11}(L), \ldots, b_{KK}(L)]$, $b_{jj}(L) = 1 - b_{jj,1}L - \cdots - b_{jj,q_j}L^{q_j}$, when $B(L)$ is in MA diagonal form. Then, when the model is in diagonal MA form, we can write the parameters of the VARMA model as a vector $\gamma = [\gamma_1, \gamma_2]'$ where $\gamma_1$ contains the AR parameters and $\gamma_2$ the MA parameters, as follows:

$$\gamma_1 = [a_{1\bullet,1}, \ldots, a_{1\bullet,p}, \ldots, a_{K\bullet,1}, \ldots, a_{K\bullet,p}], \tag{5.2}$$

$$\gamma_2 = [b_{11,1}, \ldots, b_{11,q_1}, \ldots, b_{KK,1}, \ldots, b_{KK,q_K}]. \tag{5.3}$$

The estimation method involves three steps.

**Step 1.** Estimate a VAR$(n_T)$ model by least squares, where $n_T < T/(2K)$, and compute the residuals:

$$\hat{U}_t = Y_t - \sum_{l=1}^{n_T} \hat{\Pi}_l(n_T)Y_{t-l}. \tag{5.4}$$

**Step 2.** From the residuals of step 1, compute $\hat{\Sigma}_U = \frac{1}{T}\sum_{t=n_T+1}^{T} \hat{U}_t\hat{U}_t'$, *i.e.* the corresponding estimate of the covariance matrix of $U_t$, and apply GLS to the multivariate regression

$$A(L)Y_t = [B(L) - I_K]\hat{U}_t + e_t \tag{5.5}$$

to get estimates $\tilde{A}(L)$ and $\tilde{B}(L)$. The estimator is

$$\hat{\gamma} = \Big[\sum_{t=l}^{T} \hat{Z}'_{t-1}\hat{\Sigma}_U^{-1}\hat{Z}_{t-1}\Big]^{-1}\sum_{t=l}^{T}\hat{Z}'_{t-1}\hat{\Sigma}_U^{-1}Y_t \tag{5.6}$$

with $l = n_T + \max(p,q) + 1$. Setting

$$\mathbf{Y}_{t-1}(p) = [y_{1,t-1}, \ldots, y_{K,t-1}, \ldots, y_{1,t-p}, \ldots, y_{K,t-p}], \tag{5.7}$$

$$\hat{\mathbf{U}}_{t-1} = [\hat{u}_{1,t-1}, \ldots, \hat{u}_{K,t-1}, \ldots, \hat{u}_{1,t-q}, \ldots, \hat{u}_{K,t-q}], \ \hat{\mathbf{u}}_{k,t-1} = \left[\hat{u}_{k,t-1}, \ldots, \hat{u}_{k,t-q_k}\right], \tag{5.8}$$

the matrix $\hat{Z}_{t-1}$ is defined as

$$\hat{Z}_{t-1} = \begin{bmatrix} \mathbf{Y}_{t-1}(p) & \cdots & 0 & \hat{\mathbf{u}}_{1,t-1} & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{Y}_{t-1}(p) & 0 & \cdots & \hat{\mathbf{u}}_{K,t-1} \end{bmatrix}.$$

**Step 3.** Using the second step estimates, form new residuals

$$\tilde{U}_t = Y_t - \sum_{i=1}^{p} \tilde{A}_i Y_{t-i} + \sum_{j=1}^{q} \tilde{B}_j \tilde{U}_{t-j}$$

with $\tilde{U}_t = 0$ for $t \leq \max(p, q)$, and define

$$X_t = \sum_{j=1}^{q} \tilde{B}_j X_{t-j} + Y_t \,,\; W_t = \sum_{j=1}^{q} \tilde{B}_j W_{t-j} + \tilde{U}_t \,,\; \tilde{V}_t = \sum_{j=1}^{q} \tilde{B}_j \tilde{V}_{t-j} + \tilde{Z}_t \,,$$

where $X_t = W_t = 0$ for $t \leq \max(p, q)$, and $\tilde{Z}_t$ is defined like $\hat{Z}_t$ in step 2, with $\hat{U}_t$ replaced by $\tilde{U}_t$. Then, compute a new estimate of $\Sigma_U$, $\hat{\tilde{\Sigma}}_U = \frac{1}{T}\sum_{t=\max(p,q)+1}^{T} \tilde{U}_t \tilde{U}_t'$, and regress by GLS $\tilde{U}_t + X_t - W_t$ on $\tilde{V}_{t-1}$ to obtain the following estimate of $\gamma$:

$$\hat{\gamma} = \left[ \sum_{t=\max(p,q)+1}^{T} \tilde{V}_{t-1}' \tilde{\Sigma}_U^{-1} \tilde{V}_{t-1} \right]^{-1} \left[ \sum_{t=\max(p,q)+1}^{T} \tilde{V}_{t-1}' \tilde{\Sigma}_U^{-1} \left[ \tilde{U}_t + X_t - W_t \right] \right]. \tag{5.9}$$

The consistency and asymptotic normality of the above estimators are established in Dufour and Pelletier (2013). In the previous steps, the orders of the AR and MA operators are taken as known. In practice, they are usually estimated by statistical methods or suggested by theory. Dufour and Pelletier (2013) propose an information criterion to be applied in the second step of the estimation procedure. For all $p_i \leq P$ and $q_i \leq Q$ compute

$$\log[\det(\tilde{\Sigma}_U)] + \dim(\gamma) \frac{(\log T)^{1+\delta}}{T}, \quad \delta > 0 \,. \tag{5.10}$$

Choose $\hat{p}_i$ and $\hat{q}_i$ as the set which minimizes the information criteria (5.10). The properties of estimators $\hat{p}_i$ and $\hat{q}_i$ are given in the paper.

# 6. FORECASTING

In this section, we study whether the introduction of VARMA factors can improve forecasting. We consider a simplified version of the static model (4.7) - (4.9) where $F_t$ is scalar:

$$\begin{aligned} X_{it} &= \lambda_i F_t + u_{it} \,, & (6.1)\\ u_{it} &= \delta_i u_{it-1} + \nu_{it} \,,\; i, \ldots, N, & (6.2)\\ F_t &= \phi F_{t-1} + \eta_t - \theta \eta_{t-1}. & (6.3) \end{aligned}$$

On replacing $F_t$ and $u_{it}$ in the observation equation (6.1) with the expressions in (6.2) - (6.3), we get the following forecast equation for $X_{i,T+1}$ based on the information available at time $T$:

$$X_{i,T+1|T} = \delta_i X_{iT} + \lambda_i(\phi - \delta_i) F_T - \lambda_i \theta \eta_T \,.$$

Suppose $\lambda_i \neq 0$, *i.e.* there is indeed a factor structure applicable to $X_{it}$, and $\phi \neq \delta_i$, *i.e.* the common and specific components do not have the same dynamics. With the additional assumption that $F_t$ follows an $\mathrm{AR}(1)$ process, Boivin and Ng (2005) show that taking into account the factor $F_t$ allows one to obtain better forecasts of $X_{it}$ [in terms of the mean squared error (MSE)]. We allow here for an MA component in the dynamic process of $F_t$, which provides a parsimonious way of representing an infinite-order AR structure for the factor.

Forecast performance depends on the way factors are estimated as well as the choice of forecasting model. Boivin and Ng (2005) consider static and dynamic factor estimation along with three types of forecast equations: (1) *unrestricted*, where $X_{i,T+h}$ is predicted using $X_{iT}$, $F_T$ and their lags; (2) *direct*, where $F_{T+h}$ is first predicted using its dynamic process, a forecast then used to predict $X_{i,T+h}$ with the factor equation (6.3); (3) *nonparametric*, where no parametric assumption is made on factor dynamics and its relationship with observables. The simulation and empirical results of Boivin and Ng (2005) show that the unrestricted forecast equation with static factors generally yields the best performance in terms of MSE.

## 6.1. Forecasting models

A popular way to evaluate the predictive power of a model is to conduct an out-of-sample forecasting exercise. Here, we compare the FAVARMA approach with common factor-based methods. The forecast equations are divided in two categories. First, we consider methods where no explicit dynamic factor model is used, such as diffusion-index (DI) and diffusion autoregressive (DI-AR) models [Stock and Watson (2002b)]:

$$X_{i,T+h|T} = \alpha^{(h)} + \sum_{j=1}^{m} \beta_{ij}^{(h)} F_{T-j+1} + \sum_{j=1}^{p} \rho_{ij}^{(h)} X_{i,T-j+1} \, .$$

In this case, three variants are studied: (1) “unrestricted” (with $m \geq 1$ and $p \geq 0$); (2) “DI” (with $m = 1$ and $p = 0$); (3) “DI-AR” (with $m = 1$). Second, we consider two-step methods where common and specific components are first predicted from their estimated dynamic processes, and then combined to forecast the variables of interest using the estimated observation equation. Moreover, we distinguish between sequential (or iterative) and direct methods to calculate forecasts [see Marcellino, Stock and Watson (2006) for details]:

$$X_{i,T+h|T} = \lambda_i' F_{T+h|T} + u_{i,T+h|T}$$

where $u_{i,T+h|T}$ is obtained after fitting an $\mathrm{AR}(p)$ process on $u_{it}$, while the factor forecasts are obtained using “sequential” $[F_{T+h|T} = \hat{\Phi}_{T+h-1}(L) F_{T+h-1|T}\,]$ or “direct” methods $[F_{T+h|T} = \hat{\Phi}_T^{(h)}(L) F_T\,]$.

In this exercise, the factors are defined as principal components of $X_t$. Thus, only the second type of forecast method is affected by allowing for VARMA factors. We consider four identified VARMA forms labeled: “Diag MA”, “Diag AR”, “Final MA” and “Final AR”. The FAVARMA forecasting equations have the form:

$$X_{i,T+h|T} = \lambda_i' F_{T+h|T} + u_{i,T+h|T}\ , \quad F_{T+h|T} = \hat{\Phi}_{T+h-1}(L) F_{T+h-1|T} + \hat{\Theta}_{T+h-1}(L)\eta_{T+h-1|T}\ .$$

Our benchmark forecasting model is an AR($p$) model, as in Stock and Watson (2002b) and Boivin and Ng (2005). However, given the postulated factor structure, a finite-order autoregressive model is only an approximation of the process of $X_{it}$. From Theorem **3.1**, the marginal process for each element of $X_t$ typically has an ARMA form. If the MA polynomial has roots close to the non-invertibility region, a long autoregressive model may be needed to approximate the process. For this reason, we also consider ARMA models as benchmarks, to see how they fare with respect to AR and factor-based models.

### 6.2. Monte-Carlo simulations

To assess the performance of our approach, we performed a Monte Carlo simulation comparing the forecasts of FAVARMA models (in four identified forms) with those of FAVAR models. The data were simulated using a static factor model with MA(1) factors and idiosyncratic components similar to the ones considered by Boivin and Ng (2005) and Onatski (2009b):

$$X_{it} = \lambda_i F_t + u_{it}\,,\ F_t = \eta_t - B\eta_{t-1}\,,$$

$$u_{it} = \rho_N u_{i-1,t} + \xi_{it}\,,\ \xi_{it} = \rho_T \xi_{i,t-1} + \varepsilon_{it}\,, \quad \varepsilon_{it} \sim N(0,1)\,,\ i = 1,\ldots,N,\ t = 1,\ldots,T,$$

where $\eta_t \overset{iid}{\sim} N(0,1)$, $\rho_N \in \{0.1, 0.5, 0.9\}$ determines the cross-sectional dependence, $\rho_T \in \{0.1, 0.9\}$ the time dependence, the number of factors is 2, $B = \mathrm{diag}[0.5,\ 0.3]$, $N = \{50, 100, 130\}$, and $T \in \{50, 100, 600\}$. VARMA orders are estimated as in Dufour and Pelletier (2013), the AR order for idiosyncratic component is 1, and the lag order in VAR approximation of factors dynamics is set to 6.

The results from this simulation exercise are presented in Appendix (Table 1). The numbers represent the MSE of four FAVARMA identified forms over the MSE of FAVAR direct forecasting models. When the number of time periods is small ($T = 50$), FAVARMA models strongly outperform FAVAR models, especially at long horizons. The huge improvement at horizons 24 and 36 is due to the small sample size. When compared to the iterative FAVAR model (not reported), FAVARMA models still produce better forecasts in terms of MSE, but the improvement is smaller relative to the multi-step-ahead VAR-based forecasts. When the number of time periods increases ($T = 100$, 600), the improvement of VARMA-based models is moderate, but the latter still yield better forecasts, especially at longer horizons. Another observation of interest is that FAVARMA models perform better when the factor structure is weak, *i.e.* in cases where the cross-section size is relatively small ($N = 50$ compared to $N = 100$) and idiosyncratic components are correlated.

We performed additional simulation exercises (not reported), which also demonstrate a better performance of FAVARMA-based forecasts when the number of factors increases. The description and results are available in the appendix.

# 7. APPLICATION: FORECASTING U.S. MACROECONOMIC AGGREGATES

In this section, we present an out-of-sample forecasting exercise using a balanced monthly panel from Boivin et al. (2009) which contains 128 monthly U.S. economic and financial indicators observed from 1959M01 to 2008M12. The series were initially transformed to induce stationarity.

The MSE results relative to benchmark AR$(p)$ models are presented in Table 1. The out-of-sample evaluation period is 1988M01-2008M12. In the forecasting models "unrestricted", "DI", and "DI-AR", the number of factors, the number of lags for both factors and $X_{it}$ are estimated with BIC, and are allowed to vary over the whole evaluation period. For "unrestricted" model the number of factors is 3, $m = 1$ and $p = 0$. In the case of "DI-AR" and "DI", 6 factors are used, plus 5 lags of $X_{it}$ within "DI-AR" representation.

In the FAVAR and FAVARMA models, the number of factors is set to 4. For all evaluation periods and forecasting horizons the estimated VARMA orders (AR and MA respectively) are low: 1 and $[1,1,1,1]$ for DMA form, $[1,2,1,1]$ and 1 for DAR, 1 and 2 for FMA, and $[2-4]$ and 1 for FAR form. The estimated VAR order is most of the time equal to 2, while the lag order of each idiosyncratic AR$(p)$ process is between 1 and 3. In robustness analysis, the VAR order has been set to 4, 6 and 12, but the results did not change substantially. Both univariate ARMA orders are estimated to 1, while the number of lags in the benchmark AR model fluctuates between 1 and 2.

The results in Table 1 show that VARMA factors improve the forecasts of key macroeconomic indicators across several horizons. For industrial production growth, the diffusion-index model exhibits the best performance at the one-month horizon, while diagonal MA and final MA FAVARMA models outperform the other methods for horizons of 2, 4 and 6 months. Finally, univariate ARMA models yield the smallest RMSE for the long-term forecasts. When forecasting employment growth, three FAVARMA forms outperform all other factor-based models for short and mid-term horizons. ARMA models still produce the smallest RMSE for most of the long-term horizons.

For CPI inflation, the DI model provides the smallest MSE at horizon 1, while the final AR FAVARMA models do a better job at horizons 2, 4 and 6. Several VARMA-based models perform the best for longer horizons (18, 24 and 48 months), while sequential and DI approaches dominate in forecasting 12 and 36 months ahead.

From Theorem **3.1**, it is easy to see that each component of $X_t$ follows a univariate ARMA process. The forecasts based on factor and univariate ARMA models are not in general equivalent, because different information sets are used. Even though multivariate models (such as factor models) use more variables, univariate ARMA models tend to be more parsimonious in practice, which may reduce estimation uncertainty. So these two modelling strategies can produce quite different forecasts. In Table 2 we present MSE of all factor model predictions relative to ARMA forecasts. Boldface numbers highlight cases where the ARMA model outperforms the factor-based alternatives in terms of MSE.

For industrial production, ARMA specifications do better than all diffusion-index and FAVAR models (except at the one-month horizon). For employment, the conclusion is quite similar relative to FAVARMA, while diffusion-index models perform better than ARMA at horizons 1, 2, 4, and 48. Finally, in the case of CPI inflation, ARMA model seem to be a better choice for most of

Table 1: RMSE relative to direct AR($p$) forecasts

| Industrial production growth rate: total | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Horizon | Unrestricted | DI | DI AR | Direct | Sequential | Diag MA | Diag AR | Final MA | Final AR | ARMA |
| 1 | 0.8706 | **0.8457** | 0.8958 | 0.9443 | 0.9443 | 0.8971 | 0.9019 | 0.9132 | 0.8985 | 0.9700 |
| 2 | 1.0490 | 0.9938 | 1.0106 | 1.0157 | 1.0665 | **0.9074** | 0.9202 | 0.9112 | 0.9123 | 1.0026 |
| 4 | 1.1934 | 1.0411 | 1.0527 | 1.0711 | 1.2214 | **0.8947** | 0.9906 | 0.8970 | 0.9481 | 0.9710 |
| 6 | 1.1496 | 1.0238 | 1.0245 | 1.1743 | 1.3528 | 0.9248 | 1.0494 | **0.9202** | 0.9847 | 0.9918 |
| 12 | 1.2486 | 1.0445 | 1.0389 | 1.0933 | 1.3682 | 1.0008 | 1.2215 | 1.0075 | 1.0371 | **0.9713** |
| 18 | 1.0507 | 1.0048 | 1.0207 | 1.0662 | 1.2508 | 1.0511 | 1.5098 | 1.0615 | 1.1206 | **0.9910** |
| 24 | 1.0393 | 1.0628 | 1.0748 | 1.0128 | 1.0863 | 0.9858 | 1.7920 | 0.9959 | 1.1061 | **0.9604** |
| 36 | 1.0092 | 1.0906 | 1.1437 | 1.2364 | 1.0421 | 0.9855 | 3.0304 | 0.9883 | 1.1795 | **0.9826** |
| 48 | 1.0147 | 1.1110 | 1.1212 | 1.1063 | 1.0355 | 0.9921 | 5.5321 | 0.9922 | 1.1681 | **0.9856** |
| Civilian labor force growth rate: employed. total | | | | | | | | | | |
| Horizon | Unrestricted | DI | DI AR | Direct | Sequential | Diag MA | Diag AR | Final MA | Final AR | ARMA |
| 1 | 0.8264 | 0.8832 | 0.8451 | 0.8202 | 0.8202 | **0.8004** | 0.8075 | 0.8027 | 0.8008 | 1.0496 |
| 2 | 0.9407 | 0.9391 | 0.9381 | 0.9477 | 0.9591 | 0.8931 | **0.8805** | 0.8961 | 0.8852 | 1.0422 |
| 4 | 0.9766 | 0.9739 | 0.9937 | 1.0204 | 1.0551 | 0.9213 | 0.8997 | 0.9200 | **0.8991** | 0.9993 |
| 6 | 1.0776 | 1.0799 | 1.0937 | 1.0714 | 1.1550 | 0.9667 | 0.9526 | 0.9636 | **0.9455** | 1.0032 |
| 12 | 1.0741 | 1.0742 | 1.0722 | 1.0137 | 1.1654 | 0.9718 | 0.9912 | 0.9704 | 0.9558 | **0.9507** |
| 18 | 1.0471 | 1.0488 | 1.0472 | 0.9735 | 1.1391 | 1.0073 | 1.1386 | 1.0096 | 1.0391 | **0.9721** |
| 24 | 1.0237 | 1.0580 | 1.0268 | **0.9641** | 1.1002 | 1.0154 | 1.2806 | 1.0177 | 1.0856 | 0.9893 |
| 36 | 0.9573 | 0.9099 | 0.9703 | 0.9507 | 0.9477 | 0.9070 | 1.5452 | 0.9043 | 1.0098 | **0.8957** |
| 48 | **0.9227** | 0.9236 | 0.9250 | 0.9576 | 0.9989 | 0.9652 | 2.4022 | 0.9624 | 1.0482 | 0.9550 |
| Consumer price index growth rate: all items | | | | | | | | | | |
| Horizon | Unrestricted | DI | DI AR | Direct | Sequential | Diag MA | Diag AR | Final MA | Final AR | ARMA |
| 1 | 0.8806 | **0.8700** | **0.8700** | 0.9228 | 0.9228 | 0.9144 | 0.9432 | 0.8856 | 0.9072 | 1.0143 |
| 2 | 0.9866 | 0.9942 | 0.9942 | 0.9612 | 0.9730 | 0.9309 | 0.9427 | 0.9274 | **0.9170** | 0.9856 |
| 4 | 1.0656 | 1.0732 | 1.0732 | 1.0398 | 1.0170 | 1.0007 | 1.0665 | 0.9895 | **0.9792** | 1.0129 |
| 6 | 1.1343 | 1.1334 | 1.1334 | 1.0349 | 1.0101 | 0.9946 | 1.0752 | 0.9939 | **0.9928** | 1.0364 |
| 12 | 1.1173 | 1.1279 | 1.1279 | 1.0821 | **0.9513** | 0.9572 | 1.1958 | 0.9553 | 1.0408 | 1.0297 |
| 18 | 1.0311 | 1.0379 | 1.0379 | 1.0430 | 0.9654 | **0.8894** | 1.1021 | 0.8909 | 0.9673 | 0.9391 |
| 24 | 0.9644 | 1.0712 | 1.0712 | 0.9510 | 0.9980 | 0.8819 | 1.1851 | **0.8791** | 0.9713 | 0.8805 |
| 36 | 0.7645 | **0.7627** | **0.7627** | 0.9870 | 0.9470 | 0.8329 | 1.4591 | 0.8385 | 0.9126 | 0.8619 |
| 48 | 0.8663 | 0.8488 | 0.8488 | 0.9361 | 0.9536 | **0.8292** | 2.2640 | 0.8335 | 0.8864 | 0.8511 |

Note – The numbers in bold character present the model producing the best forecasts in terms of MSE.

the horizons relatively to diffusion-index and FAVAR alternatives. On the other hand, FAVARMA models do much better, *e.g.* the final MA form beats the ARMA models at all horizons.

Based on these results, ARMA models appears to be a very good alternative to standard factor-based models at long horizons. This is not surprising since ARMA models are very parsimonious. However, FAVARMA models outperform ARMA models in most cases.

It is also of interest to see more directly how FAVARMA forecasts compare to those from FAVAR models. In Table 3, we present MSE of FAVARMA forecasting models relative to Direct and Sequential FAVAR specifications. The numbers in bold character present cases where the FAVARMA model performs better than the FAVAR.

Most numbers in Table 3 are boldfaced, *i.e.* FAVARMA models outperform standard FAVAR specifications at most horizons. This is especially the case for industrial production, where both MA VARMA forms produce smaller MSE at all horizons. At best, the FAVARMA model improves the forecasting accuracy by 32% at horizon 12. In the case of Civilian labor force, VARMA factors do improve the predicting power, but the Direct FAVAR model performs better for longer horizons.

Table 2: RMSE relative to ARMA$(p, q)$ forecasts

| Horizon | Unrestricted | DI | DI AR | Direct | Sequential | Diag MA | Diag AR | Final MA | Final AR |
|---|---|---|---|---|---|---|---|---|---|
| Industrial production growth rate: total | | | | | | | | | |
| 1 | 0.8975 | 0.8719 | 0.9235 | 0.9735 | 0.9735 | 0.9248 | 0.9298 | 0.9414 | 0.9263 |
| 2 | **1.0463** | 0.9912 | **1.0080** | **1.0131** | **1.0637** | 0.9050 | 0.9178 | 0.9088 | 0.9099 |
| 4 | **1.2290** | **1.0722** | **1.0841** | **1.1031** | **1.2579** | 0.9214 | **1.0202** | 0.9238 | 0.9764 |
| 6 | **1.1591** | **1.0323** | **1.0330** | **1.1840** | **1.3640** | 0.9324 | **1.0581** | 0.9278 | 0.9928 |
| 12 | **1.2855** | **1.0754** | **1.0696** | **1.1256** | **1.4086** | **1.0304** | **1.2576** | **1.0373** | **1.0677** |
| 18 | **1.0602** | **1.0139** | **1.0300** | **1.0759** | **1.2622** | **1.0606** | **1.5235** | **1.0711** | **1.1308** |
| 24 | **1.0822** | **1.1066** | **1.1191** | **1.0546** | **1.1311** | **1.0264** | **1.8659** | **1.0370** | **1.1517** |
| 36 | **1.0271** | **1.1099** | **1.1640** | **1.2583** | **1.0606** | **1.0030** | **3.0841** | **1.0058** | **1.2004** |
| 48 | **1.0295** | **1.1272** | **1.1376** | **1.1225** | **1.0506** | **1.0066** | **5.6129** | **1.0067** | **1.1852** |
| Civilian labor force growth rate: employed. total | | | | | | | | | |
| Horizon | Unrestricted | DI | DI AR | Direct | Sequential | Diag MA | Diag AR | Final MA | Final AR |
| 1 | 0.7873 | 0.8415 | 0.8052 | 0.7814 | 0.7814 | 0.7626 | 0.7693 | 0.7648 | 0.7630 |
| 2 | 0.9026 | 0.9011 | 0.9001 | 0.9093 | 0.9203 | 0.8569 | 0.8448 | 0.8598 | 0.8494 |
| 4 | 0.9773 | 0.9746 | 0.9944 | **1.0211** | **1.0558** | 0.9219 | 0.9003 | 0.9206 | 0.8997 |
| 6 | **1.0742** | **1.0765** | **1.0902** | **1.0680** | **1.1513** | 0.9636 | 0.9496 | 0.9605 | 0.9425 |
| 12 | **1.1298** | **1.1299** | **1.1278** | **1.0663** | **1.2258** | **1.0222** | **1.0426** | **1.0207** | **1.0054** |
| 18 | **1.0772** | **1.0789** | **1.0773** | **1.0014** | **1.1718** | **1.0362** | **1.1713** | **1.0386** | **1.0689** |
| 24 | **1.0348** | **1.0694** | **1.0379** | 0.9745 | **1.1121** | **1.0264** | **1.2945** | **1.0287** | **1.0973** |
| 36 | **1.0688** | **1.0159** | **1.0833** | **1.0614** | **1.0581** | **1.0126** | **1.7251** | **1.0096** | **1.1274** |
| 48 | 0.9662 | 0.9671 | 0.9686 | **1.0027** | **1.0460** | **1.0107** | **2.5154** | **1.0077** | **1.0976** |
| Consumer price index growth rate: all items | | | | | | | | | |
| Horizon | Unrestricted | DI | DI AR | Direct | Sequential | Diag MA | Diag AR | Final MA | Final AR |
| 1 | 0.8682 | 0.8577 | 0.8577 | 0.9098 | 0.9098 | 0.9015 | 0.9299 | 0.8731 | 0.8944 |
| 2 | **1.0010** | **1.0087** | **1.0087** | 0.9752 | 0.9872 | 0.9445 | 0.9565 | 0.9409 | 0.9304 |
| 4 | **1.0520** | **1.0595** | **1.0595** | **1.0266** | **1.0040** | 0.9880 | **1.0529** | 0.9769 | 0.9667 |
| 6 | **1.0945** | **1.0936** | **1.0936** | 0.9986 | 0.9746 | 0.9597 | **1.0374** | 0.9590 | 0.9579 |
| 12 | **1.0851** | **1.0954** | **1.0954** | **1.0509** | 0.9239 | 0.9296 | **1.1613** | 0.9277 | **1.0108** |
| 18 | **1.0980** | **1.1052** | **1.1052** | **1.1106** | **1.0280** | 0.9471 | **1.1736** | 0.9487 | **1.0300** |
| 24 | **1.0953** | **1.2166** | **1.2166** | **1.0801** | **1.1334** | **1.0016** | **1.3459** | 0.9984 | **1.1031** |
| 36 | 0.8870 | 0.8849 | 0.8849 | **1.1451** | **1.0987** | 0.9664 | **1.6929** | 0.9729 | **1.0588** |
| 48 | **1.0179** | 0.9973 | 0.9973 | **1.0999** | **1.1204** | 0.9743 | **2.6601** | 0.9793 | **1.0415** |

Note – The numbers in bold character present cases where the ARMA model outperforms the factor-based alternatives in terms of MSE.

Finally, both diagonal and final MA FAVARMA specifications provide smaller MSEs over all horizons in predicting CPI inflation. The improvement increases with the forecast horizons, and reaches a maximum of 15%.

We performed a similar exercise with a Canadian data set from Boivin, Giannoni and Stevanović (2009b). We found that VARMA factors help in predicting several key Canadian macroeconomic aggregates, relative to standard factor models, and at many forecasting horizons. The description and results are available in the Appendix.

Table 3: MSE of FAVARMA relative to FAVAR forecasting models

| Horizon | Diag MA | Diag AR | Final MA | Final AR | Diag MA | Diag AR | Final MA | Final AR |
|---|---|---|---|---|---|---|---|---|
| | Industrial production growth rate: total | | | | | | | |
| | VARMA/Direct | | | | VARMA/Sequential | | | |
| 1 | **0.9500** | **0.9551** | **0.9671** | **0.9515** | **0.9500** | **0.9551** | **0.9671** | **0.9515** |
| 2 | **0.8934** | **0.9060** | **0.8971** | **0.8982** | **0.8508** | **0.8628** | **0.8544** | **0.8554** |
| 4 | **0.8353** | **0.9248** | **0.8375** | **0.8852** | **0.7325** | **0.8110** | **0.7344** | **0.7762** |
| 6 | **0.7875** | **0.8936** | **0.7836** | **0.8385** | **0.6836** | **0.7757** | **0.6802** | **0.7279** |
| 12 | **0.9154** | 1.1173 | **0.9215** | **0.9486** | **0.7315** | **0.8928** | **0.7364** | **0.7580** |
| 18 | **0.9858** | 1.4161 | **0.9956** | 1.0510 | **0.8403** | 1.2071 | **0.8487** | **0.8959** |
| 24 | **0.9733** | 1.7694 | **0.9833** | 1.0921 | **0.9075** | 1.6496 | **0.9168** | 1.0182 |
| 36 | **0.7971** | 2.4510 | **0.7993** | **0.9540** | **0.9457** | 2.9080 | **0.9484** | 1.1318 |
| 48 | **0.8968** | 5.0005 | **0.8969** | 1.0559 | **0.9581** | 5.3424 | **0.9582** | 1.1281 |
| | Civilian labor force growth rate: employed. total | | | | | | | |
| | VARMA/Direct | | | | VARMA/Sequential | | | |
| Horizon | Diag MA | Diag AR | Final MA | Final AR | Diag MA | Diag AR | Final MA | Final AR |
| 1 | **0.9759** | **0.9845** | **0.9787** | **0.9763** | **0.9759** | **0.9845** | **0.9787** | **0.9763** |
| 2 | **0.9424** | **0.9291** | **0.9456** | **0.9341** | **0.9312** | **0.9180** | **0.9343** | **0.9229** |
| 4 | **0.9029** | **0.8817** | **0.9016** | **0.8811** | **0.8732** | **0.8527** | **0.8720** | **0.8521** |
| 6 | **0.9023** | **0.8891** | **0.8994** | **0.8825** | **0.8370** | **0.8248** | **0.8343** | **0.8186** |
| 12 | **0.9587** | **0.9778** | **0.9573** | **0.9429** | **0.8339** | **0.8505** | **0.8327** | **0.8201** |
| 18 | 1.0347 | 1.1696 | 1.0371 | 1.0674 | **0.8843** | **0.9996** | **0.8863** | **0.9122** |
| 24 | 1.0532 | 1.3283 | 1.0556 | 1.1260 | **0.9229** | 1.1640 | **0.9250** | **0.9867** |
| 36 | **0.9540** | 1.6253 | **0.9512** | 1.0622 | **0.9571** | 1.6305 | **0.9542** | 1.0655 |
| 48 | 1.0079 | 2.5086 | 1.0050 | 1.0946 | **0.9663** | 2.4048 | **0.9635** | 1.0494 |
| | Consumer price index growth rate: all items | | | | | | | |
| | VARMA/Direct | | | | VARMA/Sequential | | | |
| Horizon | Diag MA | Diag AR | Final MA | Final AR | Diag MA | Diag AR | Final MA | Final AR |
| 1 | **0.9909** | 1.0221 | **0.9597** | 0.9831 | **0.9909** | 1.0221 | **0.9597** | **0.9831** |
| 2 | **0.9685** | **0.9808** | **0.9648** | 0.9540 | **0.9567** | **0.9689** | **0.9531** | **0.9424** |
| 4 | **0.9624** | 1.0257 | **0.9516** | 0.9417 | **0.9840** | 1.0487 | **0.9730** | **0.9628** |
| 6 | **0.9611** | 1.0389 | **0.9604** | 0.9593 | **0.9847** | 1.0644 | **0.9840** | **0.9829** |
| 12 | **0.8846** | 1.1051 | **0.8828** | 0.9618 | 1.0062 | 1.2570 | 1.0042 | 1.0941 |
| 18 | **0.8527** | 1.0567 | **0.8542** | 0.9274 | **0.9213** | 1.1416 | **0.9228** | 1.0020 |
| 24 | **0.9273** | 1.2462 | **0.9244** | 1.0213 | **0.8837** | 1.1875 | **0.8809** | **0.9732** |
| 36 | **0.8439** | 1.4783 | **0.8495** | 0.9246 | **0.8795** | 1.5408 | **0.8854** | **0.9637** |
| 48 | **0.8858** | 2.4185 | **0.8904** | 0.9469 | **0.8695** | 2.3742 | **0.8741** | **0.9295** |

Note – The numbers in bold character present cases where the FAVARMA model performs better than the FAVAR.

# 8. APPLICATION: EFFECTS OF MONETARY POLICY SHOCKS

In the recent empirical macroeconomic literature, structural factor analysis has become popular: using hundreds of observable economic indicators appears to overcome several difficulties associated with standard structural VAR modelling. In particular, bringing more information, while keeping the model parsimonious, may provide corrections for omitted and measurement errors; see Bernanke et al. (2005) and Forni, Giannone, Lippi and Reichlin (2009).

We reconsider the empirical study of Bernanke et al. (2005) with the same data, the same method to extract factors (principal components) and the same observed factor (Federal Funds Rate). So we set $D(L) = 0$ and $G = I$ in equations (4.8)-(4.9). The difference is that we estimate VARMA dynamics on static factors instead of imposing a finite-order VAR representation. The monetary policy shock is identified from the Cholesky decomposition of the residual covariance matrix in (4.9), where the observed factor is ordered last. We consider all four identified VARMA forms, but retain only the diagonal MA representation. The number of latent factors is set to five, and we estimate a VARMA (2.1) model [these orders were estimated using the information criterion in Dufour and Pelletier (2013)].

In Figure 1, we present FAVARMA$(2,1)$-based impulse responses, with 90% confidence intervals (computed from 5000 bootstrap replications). A contractionary monetary policy shock generates a significant and very persistent economic downturn. The confidence intervals are more informative than those from FAVAR models. We conclude that impulse responses from a parsimonious 6-factor FAVARMA(2, 1) model provide a precise and plausible picture of the effect and transmission of monetary policy in the U.S.

In Figure 2, we compare the impulse responses to a monetary policy shock estimated from FAVAR and FAVARMA-DMA models. The FAVAR impulse coefficients were computed for several VAR orders. To get similar responses from a standard FAVAR model, the Akaike information criterion leads to a lag order of 14. So we need to estimate 84 coefficients governing the factors dynamics in the FAVARMA framework, while the FAVAR model requires 510 VAR parameters.

The approximation of the true factor process could be important when choosing the parametric bootstrap procedure to obtain statistical inference on objects of interest. The confidence intervals are produced as follows [see Yamamoto (2011) for theoretical justification of this bootstrap procedure].
**Step 1** Shuffle the time periods, with replacement, of the residuals in (4.9) to get the bootstrap sample $\tilde{\eta}_t$. Then, resample static factors using estimated VARMA coefficients:

$$\tilde{F}_t = \hat{\Phi}(L)\tilde{F}_{t-1} + \hat{\Theta}\tilde{\eta}_t\,.$$

**Step 2** Shuffle the time periods, with replacement, of the residuals in (4.7) to get the bootstrap sample $\tilde{u}_t$. Then resample the observable series using $\tilde{F}_t$ and the estimated loadings:

$$\tilde{X}_t = \hat{\Lambda}\tilde{F}_t + \tilde{u}_t\,.$$

**Step 3** Estimate FAVARMA model on $\tilde{X}_t$, identify structural shocks and produce impulse responses.

# 9. CONCLUSION

In this paper, we have studied the relationship between VARMA and factor representations of a vector stochastic process and proposed the FAVARMA model. We started by observing that multivariate time series and their associated factors cannot in general both follow a finite-order VAR process. When the factors are obtained as linear combinations of observable series, the dynamic process of the latter has a VARMA structure, not a finite-order VAR form. In addition, even if the factors follow a finite-order VAR process, this implies a VARMA representation for the observable series. As a result, we proposed the FAVARMA framework, which combines two parsimonious methods to represent the dynamic interactions between a large number of time series: factor analysis and VARMA modeling.

To illustrate the performance of the proposed approach, we performed Monte Carlo simulations

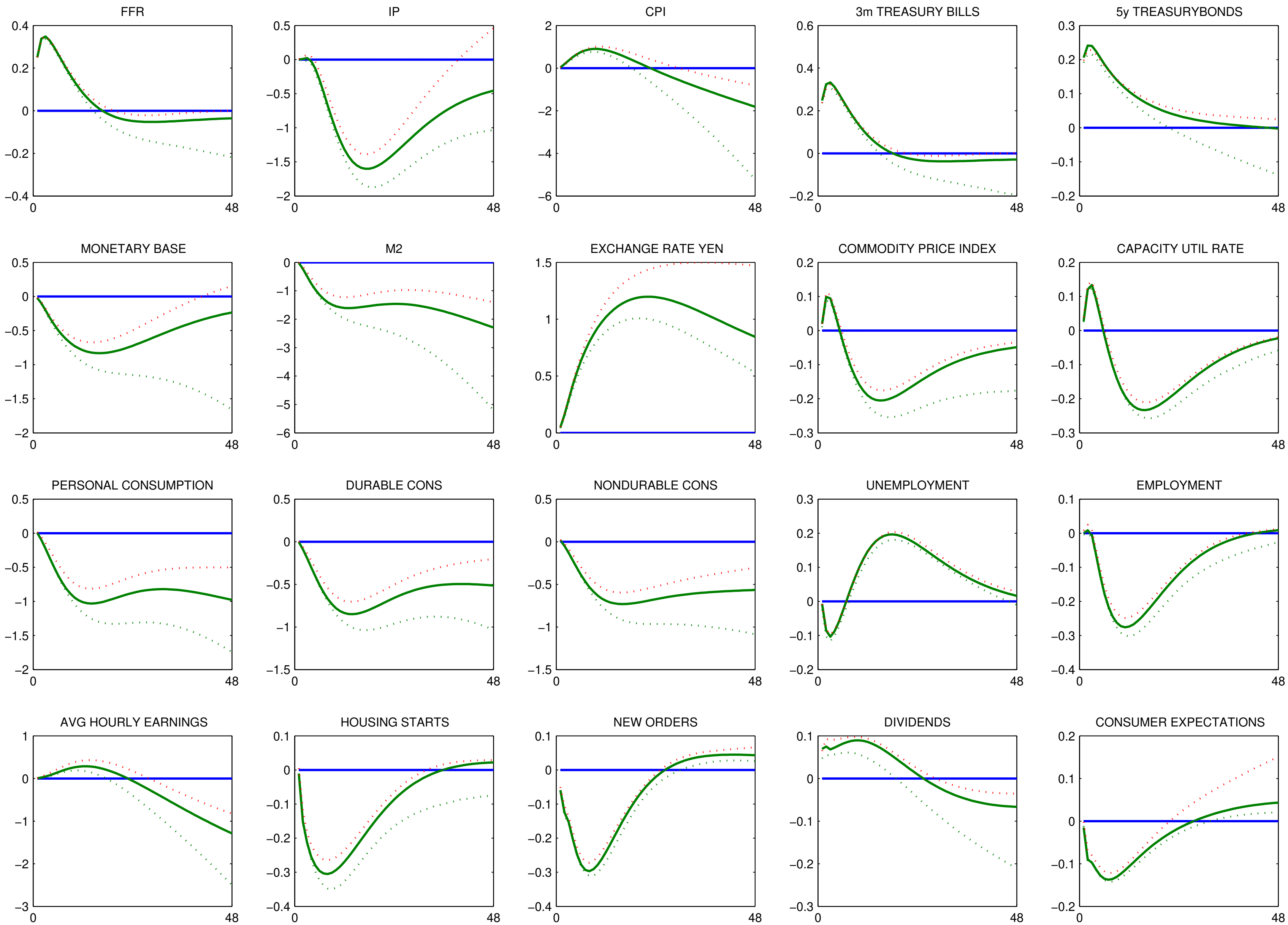


Figure 1: FAVARMA-DMA impulse responses to monetary policy shock

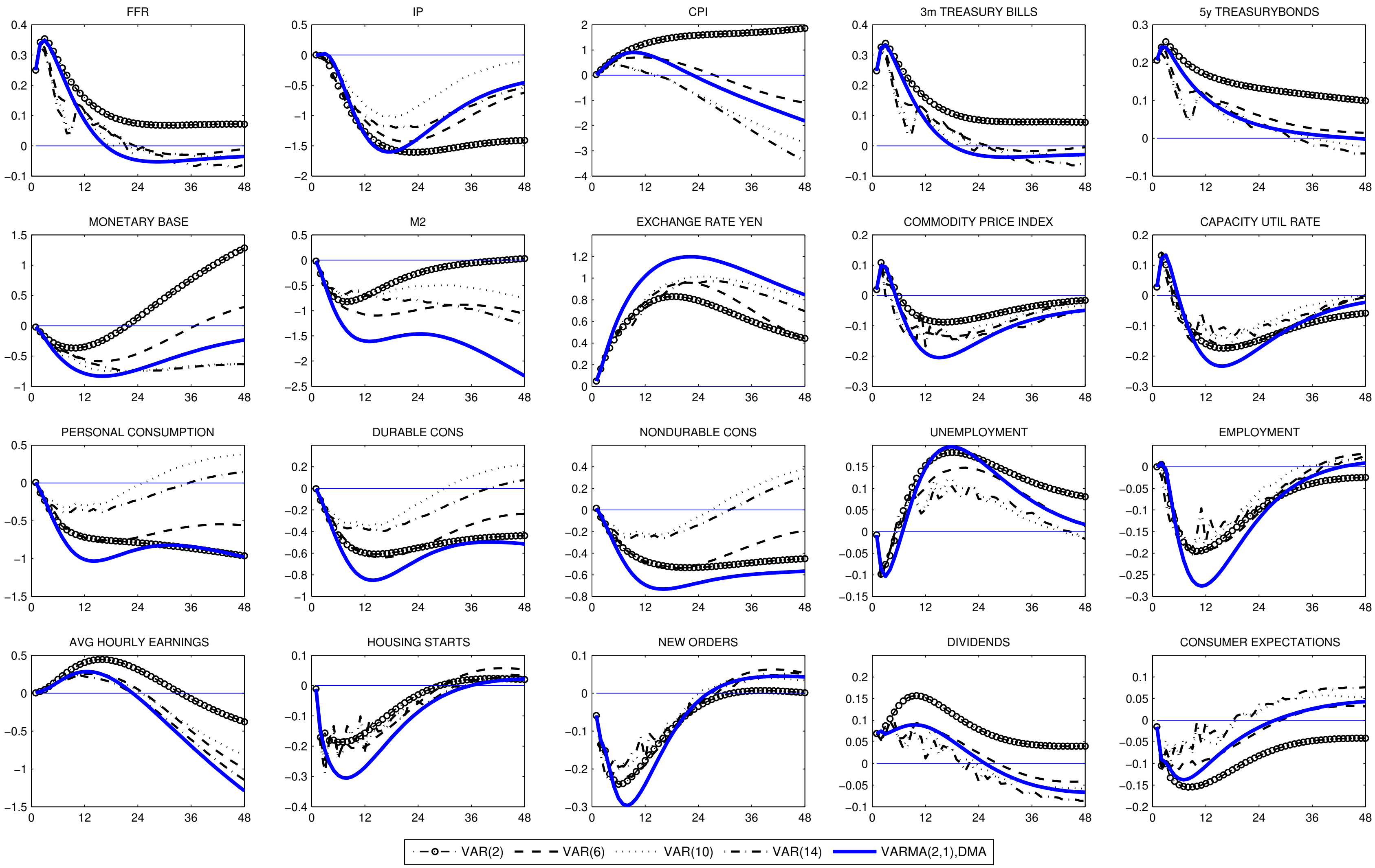


Figure 2: Comparison between FAVAR and FAVARMA impulse responses to a monetary policy shock

and found that VARMA modelling is quite helpful, especially in small samples cases – where the best improvement occurred at long horizons – but also in cases where the sample size is comparable to the one in our empirical data.

We applied our approach in an out-of-sample forecasting exercises based on a large U.S. monthly panel. The results show that VARMA factors help predict several key macroeconomic aggregates relative to standard factor models. In particular, FAVARMA models generally outperform FAVAR forecasting models, especially if we use MA VARMA-factor specifications.

Finally, we estimated the effect of monetary policy using the data and the identification scheme of Bernanke et al. (2005). We found that impulse responses from a parsimonious 6-factor FAVARMA (2.1) factor model yields a precise and plausible picture of the effect and the transmission of monetary policy in the U.S. To get similar responses from a standard FAVAR model, the Akaike information criterion leads to a lag order of 14. So we need to estimate 84 coefficients governing the factors dynamics in the FAVARMA framework, while the FAVAR model requires 510 parameters

# APPENDIX

## A. PROOFS

**Proof of Theorem 3.1** Since $\Lambda$ has full rank, we can multiply (3.1) by $(\Lambda'\Lambda)^{-1}\Lambda'$ to get

$$F_{t-1} = (\Lambda'\Lambda)^{-1}\Lambda' X_{t-1} - (\Lambda'\Lambda)^{-1}\Lambda' u_{t-1}. \tag{A.1}$$

If we now substitute $F_{t-1}$ in (3.3), we see that

$$F_t = \Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' X_{t-1} - \Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' u_{t-1} + a_t,$$

hence, on substituting the latter expression for $F_t$ in (3.1), and defining $A_1(L) = \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda'$,

$$X_t = \Lambda F_t + u_t = A_1(L)X_{t-1} + u_t - A_1(L)u_{t-1} + \Lambda a_t = A_1(L)X_{t-1} + A(L)u_t + \Lambda a_t = A_1(L)X_{t-1} + B(L)e_t$$

where $A(L) = I - A_1(L)L$ and $e_t = [u_t \vdots a_t]'$. This yields the representation (3.5).
We will now show that $X_t$ can can be written as a VARMA process where the noise is the innovation process of $X_t$. Since $X_t$ is regular strictly indeterministic weakly stationary process, it has a moving-average representation of the form (2.1) where $\varepsilon_t = X_t - P_L[X_t \,|\, X_{t-1},\, X_{t-2}, \ldots]$ and $P_L[X_t \,|\, X_{t-1},\, X_{t-2}, \ldots]$ is the best linear forecast of $X_t$ based on its own past, $\Sigma_\varepsilon = E[\varepsilon_t\varepsilon_t']$ and $\det[\Sigma_\varepsilon] > 0$. Using the assumptions (3.2) and (3.4), it is easy to see that

$$\mathsf{E}[X_{t-j}u_t'] = \mathsf{E}[X_{t-j}a_t'] = \mathsf{E}[u_t\varepsilon_{t-j}'] = \mathsf{E}[a_t\varepsilon_{t-j}'] = 0 \text{ for } j \geq 1. \tag{A.2}$$

Then

$$A(L)X_t = A(L)\Psi(L)\varepsilon_t = \bar{\Psi}(L)\varepsilon_t = \sum_{j=0}^{\infty} \bar{\Psi}_j \varepsilon_{t-j} \tag{A.3}$$

where $\bar{\Psi}_j = \sum_{i=0}^{p+1} A_i \Psi_{j-i}$ and $\Psi_s = 0$ for $s < 0$, $s = j - i$. Let us now multiply $A(L)X_t$ by $\varepsilon_{t-k}'$ and take the expected value: using (A.3) and (3.5), we get

$$\begin{aligned} \mathsf{E}[A(L)X_t\varepsilon_{t-k}'] &= \sum_{j=0}^{\infty} \bar{\Psi}_j \mathsf{E}[\varepsilon_{t-j}\varepsilon_{t-k}'] = B_j\Sigma_\varepsilon && \text{(A.4)} \\ &= \mathsf{E}\big[\big(A(L)u_t + \Lambda a_t\big)\varepsilon_{t-k}'\big] = 0 \text{ for } k > p+1, && \text{(A.5)} \end{aligned}$$

hence $\bar{\Psi}_j = 0$ for $k > p+1$, so that $X_t$ has the following VARMA$(p+1,\, p+1)$ representation:

$$A(L)X_t = \bar{\Psi}(L)\varepsilon_t \tag{A.6}$$

where $\bar{\Psi}(L) = \sum_{j=0}^{p+1} \bar{\Psi}_j L^j$ .

**Proof of Theorem 3.2** To obtain the representations of $X_t$, we follow the same steps as in the previous proof except we substitute (A.1) for $F_{t-1}$ in (3.7), which yields

$$X_t = \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' X_{t-1} + u_t - \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' u_{t-1} + \Lambda\Theta(L)a_t .$$

Defining $A(L)$ and $e_t$ as above, with $B(L) = [A(L) \vdots \Lambda\Theta(L)]$, gives the representation as in (3.5). Then, remark that (A.5) becomes

$$\mathsf{E}[\big(A(L)u_t + \Lambda\Theta(L)a_t\big)\varepsilon'_{t-k}] = 0 \text{ for } k > \max(p+1, q)\,, \tag{A.7}$$

so $X_t$ has a VARMA$(p+1, \max(p+1, q))$.

**Proof of Theorem 3.3** $F_t = CX_t$, where $C$ is a $K \times N$ full row rank matrix. Properties (i) and (ii) are easily proved using Lütkepohl (2005, Corollaries 11.1.1 and 11.1.2). For (iii), if $X_t$ has an MA representation as in (2.1) or (2.4), the result is obtained using Lütkepohl (1987, Propositions 4.1 and 4.2).

## B. SIMULATION RESULTS: FAVARMA AND FAVAR FORECASTS

Table 1 contains the results of the Monte Carlo simulation exercise presented in Section 6.2. The numbers represent the MSE of four FAVARMA identified forms over the MSE of FAVAR direct forecasting models.

Table 1: Comparison between FAVARMA and FAVAR forecasts: Monte Carlo simulations

| | $\rho_T = 0.9, \rho_N = 0.5$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $T = 50, N = 50$ | | | | $T = 50, N = 100$ | | | |
| Horizon | Diag MA | Diag AR | Final MA | Final AR | Diag MA | Diag AR | Final MA | Final AR |
| 1 | 1.0078 | 1.1405 | 0.9235 | 1.3858 | 1.0061 | 1.0945 | 0.9084 | 1.4722 |
| 2 | 1.0199 | 1.0852 | 0.9483 | 1.3189 | 1.0302 | 1.0762 | 0.9383 | 1.3660 |
| 4 | 0.8872 | 0.9459 | 0.8350 | 1.0746 | 0.9338 | 1.0242 | 0.8745 | 1.1542 |
| 6 | 0.8122 | 0.9181 | 0.7635 | 0.9536 | 0.8514 | 0.9375 | 0.7954 | 1.0010 |
| 12 | 0.6311 | 0.8392 | 0.6072 | 0.7198 | 0.6857 | 0.9278 | 0.6533 | 0.8036 |
| 18 | 0.4913 | 0.7186 | 0.4754 | 0.5339 | 0.5181 | 0.8285 | 0.4955 | 0.5744 |
| 24 | 0.3762 | 0.6192 | 0.3706 | 0.4237 | 0.3846 | 0.7215 | 0.3788 | 0.4291 |
| 36 | 0.1394 | 0.2429 | 0.1369 | 0.1480 | 0.1445 | 0.3006 | 0.1422 | 0.1560 |
| | $T = 100, N = 50$ | | | | $T = 600, N = 130$ | | | |
| 1 | 1.0761 | 1.1170 | 1.0004 | 1.6656 | 1.0130 | 1.0126 | 1.0093 | 1.0070 |
| 2 | 1.0865 | 1.1495 | 1.0193 | 1.5676 | 0.9962 | 0.9956 | 0.9952 | 0.9951 |
| 4 | 1.0537 | 1.0890 | 1.0038 | 1.4432 | 0.9945 | 0.9950 | 0.9947 | 0.9947 |
| 6 | 1.0168 | 1.0392 | 0.9686 | 1.3060 | 0.9945 | 0.9954 | 0.9946 | 0.9946 |
| 12 | 0.9183 | 0.9915 | 0.8960 | 1.2573 | 0.9871 | 0.9883 | 0.9873 | 0.9873 |
| 18 | 0.8886 | 0.9848 | 0.8552 | 1.1123 | 0.9831 | 0.9880 | 0.9832 | 0.9832 |
| 24 | 0.8643 | 0.9706 | 0.8198 | 1.1203 | 0.9831 | 0.9830 | 0.9828 | 0.9828 |
| 36 | 0.8078 | 0.9754 | 0.7956 | 1.0742 | 0.9863 | 0.9846 | 0.9847 | 0.9847 |
| | $\rho_T = 0.9, \rho_N = 0.1$ | | | | | | | |
| | $T = 50, N = 50$ | | | | $T = 50, N = 100$ | | | |
| Horizon | Diag MA | Diag AR | Final MA | Final AR | Diag MA | Diag AR | Final MA | Final AR |
| 1 | 1.0203 | 1.0656 | 0.8897 | 1.2688 | 0.9977 | 1.0303 | 0.9026 | 1.3464 |
| 2 | 0.9689 | 1.0113 | 0.8982 | 1.1708 | 1.0013 | 1.0406 | 0.9038 | 1.1735 |
| 4 | 0.9142 | 0.9508 | 0.8616 | 1.0391 | 0.9032 | 0.9166 | 0.8461 | 1.0029 |
| 6 | 0.8420 | 0.8656 | 0.7851 | 0.9213 | 0.8841 | 0.8798 | 0.8054 | 0.9182 |
| 12 | 0.6401 | 0.7487 | 0.6235 | 0.7038 | 0.7042 | 0.8089 | 0.6850 | 0.7582 |
| 18 | 0.5208 | 0.6774 | 0.5133 | 0.5609 | 0.5469 | 0.6970 | 0.5296 | 0.5742 |
| 24 | 0.4095 | 0.5979 | 0.4124 | 0.4322 | 0.4380 | 0.5724 | 0.4282 | 0.4499 |
| 36 | 0.1417 | 0.2169 | 0.1402 | 0.1447 | 0.1453 | 0.2152 | 0.1424 | 0.1535 |
| | $T = 100, N = 50$ | | | | $T = 600, N = 130$ | | | |
| 1 | 1.0622 | 1.0751 | 0.9990 | 1.3846 | 0.9978 | 0.9980 | 0.9984 | 0.9927 |
| 2 | 1.0578 | 1.0368 | 0.9913 | 1.2818 | 0.9935 | 0.9951 | 0.9935 | 0.9933 |
| 4 | 1.0254 | 1.0088 | 0.9729 | 1.2141 | 0.9890 | 0.9894 | 0.9891 | 0.9891 |
| 6 | 1.0058 | 0.9720 | 0.9477 | 1.1812 | 0.9892 | 0.9892 | 0.9892 | 0.9892 |
| 12 | 0.9480 | 0.9163 | 0.8819 | 1.0303 | 0.9919 | 0.9918 | 0.9919 | 0.9919 |
| 18 | 0.9371 | 0.9068 | 0.8823 | 1.0173 | 0.9784 | 0.9784 | 0.9784 | 0.9784 |
| 24 | 0.9441 | 0.8755 | 0.8626 | 1.0214 | 0.9807 | 0.9807 | 0.9807 | 0.9807 |
| 36 | 0.8591 | 0.8376 | 0.8013 | 0.9264 | 0.9796 | 0.9796 | 0.9796 | 0.9796 |

Table B.1: Monte Carlo simulation results (continued)

| | $\rho_T = 0.1, \rho_N = 0.9$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $T = 50, N = 50$ | | | | $T = 50, N = 100$ | | | |
| Horizon | Diag MA | Diag AR | Final MA | Final AR | Diag MA | Diag AR | Final MA | Final AR |
| 1 | 0.8978 | 0.9108 | 0.8924 | 0.9362 | 0.9329 | 0.8880 | 0.8585 | 0.9208 |
| 2 | 0.8522 | 0.8716 | 0.8606 | 0.9168 | 0.8289 | 0.8194 | 0.8228 | 0.8642 |
| 4 | 0.8381 | 0.8420 | 0.8524 | 0.8601 | 0.8195 | 0.8213 | 0.8187 | 0.8238 |
| 6 | 0.8213 | 0.8227 | 0.8225 | 0.8210 | 0.7852 | 0.7806 | 0.7799 | 0.7795 |
| 12 | 0.7923 | 0.7906 | 0.7905 | 0.7907 | 0.7630 | 0.7569 | 0.7567 | 0.7568 |
| 18 | 0.6803 | 0.6770 | 0.6771 | 0.6772 | 0.6582 | 0.6576 | 0.6577 | 0.6577 |
| 24 | 0.5367 | 0.5363 | 0.5364 | 0.5364 | 0.4865 | 0.4864 | 0.4863 | 0.4862 |
| 36 | 0.0946 | 0.0956 | 0.0944 | 0.0944 | 0.0801 | 0.0799 | 0.0799 | 0.0800 |
| | $T = 100, N = 50$ | | | | $T = 600, N = 130$ | | | |
| 1 | 0.9680 | 0.9676 | 0.9560 | 0.9515 | 0.9931 | 0.9995 | 0.9926 | 0.9921 |
| 2 | 0.9332 | 0.9304 | 0.9306 | 0.9310 | 0.9881 | 0.9929 | 0.9882 | 0.9878 |
| 4 | 0.9338 | 0.9261 | 0.9257 | 0.9257 | 0.9882 | 0.9895 | 0.9893 | 0.9894 |
| 6 | 0.9467 | 0.9350 | 0.9351 | 0.9351 | 0.9831 | 0.9831 | 0.9830 | 0.9830 |
| 12 | 0.9358 | 0.9359 | 0.9359 | 0.9359 | 0.9825 | 0.9825 | 0.9825 | 0.9825 |
| 18 | 0.9297 | 0.9298 | 0.9297 | 0.9297 | 0.9874 | 0.9873 | 0.9873 | 0.9873 |
| 24 | 0.9140 | 0.9142 | 0.9143 | 0.9143 | 0.9887 | 0.9886 | 0.9886 | 0.9886 |
| 36 | 0.9044 | 0.9047 | 0.9043 | 0.9043 | 0.9929 | 0.9930 | 0.9930 | 0.9930 |

| | $\rho_T = 0.1, \rho_N = 0.1$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $T = 50, N = 50$ | | | | $T = 50, N = 100$ | | | |
| Horizon | Diag MA | Diag AR | Final MA | Final AR | Diag MA | Diag AR | Final MA | Final AR |
| 1 | 0.9439 | 0.8761 | 0.7969 | 0.9618 | 0.9289 | 0.8919 | 0.8155 | 0.9675 |
| 2 | 0.8029 | 0.7863 | 0.7764 | 0.8459 | 0.7888 | 0.7900 | 0.7736 | 0.8569 |
| 4 | 0.7894 | 0.7542 | 0.7533 | 0.7742 | 0.7513 | 0.7533 | 0.7525 | 0.7687 |
| 6 | 0.7580 | 0.7420 | 0.7409 | 0.7438 | 0.7477 | 0.7488 | 0.7446 | 0.7458 |
| 12 | 0.6773 | 0.6751 | 0.6751 | 0.6754 | 0.6575 | 0.6604 | 0.6560 | 0.6613 |
| 18 | 0.5757 | 0.5700 | 0.5701 | 0.5761 | 0.5741 | 0.5753 | 0.5704 | 0.5728 |
| 24 | 0.4106 | 0.4074 | 0.4073 | 0.4084 | 0.4329 | 0.4303 | 0.4304 | 0.4317 |
| 36 | 0.0726 | 0.0721 | 0.0721 | 0.0721 | 0.0719 | 0.0722 | 0.0721 | 0.0721 |
| | $T = 100, N = 50$ | | | | $T = 600, N = 130$ | | | |
| 1 | 0.9702 | 0.9672 | 0.9290 | 0.9316 | 0.9838 | 0.9874 | 0.9868 | 0.9840 |
| 2 | 0.8998 | 0.9053 | 0.8985 | 0.8993 | 0.9816 | 0.9904 | 0.9811 | 0.9811 |
| 4 | 0.9095 | 0.9003 | 0.9000 | 0.8997 | 0.9891 | 0.9894 | 0.9891 | 0.9891 |
| 6 | 0.8806 | 0.8771 | 0.8767 | 0.8767 | 0.9821 | 0.9822 | 0.9821 | 0.9821 |
| 12 | 0.8855 | 0.8841 | 0.8839 | 0.8839 | 0.9778 | 0.9778 | 0.9778 | 0.9778 |
| 18 | 0.8725 | 0.8704 | 0.8702 | 0.8702 | 0.9852 | 0.9852 | 0.9852 | 0.9852 |
| 24 | 0.8711 | 0.8707 | 0.8709 | 0.8709 | 0.9815 | 0.9815 | 0.9815 | 0.9815 |
| 36 | 0.8183 | 0.8185 | 0.8183 | 0.8183 | 0.9790 | 0.9790 | 0.9790 | 0.9790 |

## C. SIMULATION RESULTS: DIFFERENT FACTOR NUMBERS

The simulation exercise in this section studies how FAVARMA-based forecasts have a performance when the number of factors increases. shows that FAVARMA-based forecasts have a performance when the number of factors increases. The simulation design is described in following:

- time dimension: $T = 100$;
- cross-section dimension: $N = 100$;
- number of factors: $K \in \{3,4,6\}$;
- idiosyncratic component dynamics: $u_{it} = \kappa v_{it}$, $v_{it} \sim N(0, \sigma^2_{v_i})$ such that the common component explains a fraction $\vartheta$ of the variance of $X_t$; following Boivin and Ng (2005), $\vartheta$ is set to 0.5 while for the first series in panel $X_t$, the one that is forecasted: $\text{var}(\lambda_1 F_t)/\text{var}(X_{1t}) = 0.75$;
- MA coefficients matrices:
  - $K = 3$
$$B = \begin{bmatrix} 0.2350 & 0 & 0 \\ 0 & 0.2317 & 0 \\ 0 & 0 & 0.5776 \end{bmatrix}$$
  - $K = 4$
$$B = \begin{bmatrix} 0.3365 & 0 & 0 & 0 \\ 0 & 0.2420 & 0 & 0 \\ 0 & 0 & 0.0610 & 0 \\ 0 & 0 & 0 & 0.4735 \end{bmatrix}$$
  - $K = 6$
$$B = \begin{bmatrix} 0.1558 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.4827 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4525 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5320 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6604 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2763 \end{bmatrix}$$
- VAR order: 4;
- VARMA orders: estimated as in Dufour and Pelletier (2013);
- AR order for idiosyncratic component: 1.

The results are presented in Table 2 and demonstrate that FAVARMA-based forecasts have a better performance as the number of factors increases.

Table 2: Comparison between FAVARMA and FAVAR forecasts for different factor numbers
Monte Carlo simulations

| | RELATIVE MSE TO FAVAR(4) DIRECT MODEL | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $K=3$ | | | | $K=4$ | | | | $K=6$ | | | |
| Horizon | Diag MA | Diag AR | Final MA | Final AR | Diag MA | Diag AR | Final MA | Final AR | Diag MA | Diag AR | Final MA | Final AR |
| 1 | 0.9638 | 0.9643 | 0.9285 | 0.9330 | 0.9194 | 0.9182 | 0.8866 | 0.8927 | 0.7282 | 0.6615 | 0.6905 | 0.6907 |
| 2 | 0.9085 | 0.9174 | 0.9076 | 0.9133 | 0.8792 | 0.8901 | 0.8805 | 0.8866 | 0.8261 | 0.8615 | 0.8244 | 0.8385 |
| 4 | 0.8971 | 0.8966 | 0.8965 | 0.8961 | 0.8764 | 0.8775 | 0.8764 | 0.8769 | 0.8030 | 0.8030 | 0.8010 | 0.8072 |
| 6 | 0.9038 | 0.9037 | 0.9035 | 0.9036 | 0.8548 | 0.8549 | 0.8548 | 0.8549 | 0.9182 | 0.9180 | 0.9182 | 0.9204 |
| 12 | 0.8808 | 0.8807 | 0.8807 | 0.8807 | 0.8416 | 0.8418 | 0.8418 | 0.8418 | 0.7983 | 0.7997 | 0.7983 | 0.7983 |
| 18 | 0.8831 | 0.8831 | 0.8831 | 0.8831 | 0.8455 | 0.8454 | 0.8454 | 0.8454 | 0.9393 | 0.9383 | 0.9393 | 0.9393 |
| 24 | 0.8757 | 0.8756 | 0.8756 | 0.8756 | 0.8425 | 0.8425 | 0.8425 | 0.8425 | 0.7287 | 0.7286 | 0.7287 | 0.7287 |
| 36 | 0.8344 | 0.8343 | 0.8343 | 0.8343 | 0.7930 | 0.7932 | 0.7932 | 0.7932 | 0.5466 | 0.5466 | 0.5466 | 0.5466 |
| | RELATIVE MSE TO FAVAR(4) ITERATIVE MODEL | | | | | | | | | | | |
| | $K=3$ | | | | $K=4$ | | | | $K=6$ | | | |
| Horizon | Diag MA | Diag AR | Final MA | Final AR | Diag MA | Diag AR | Final MA | Final AR | Diag MA | Diag AR | Final MA | Final AR |
| 1 | 0.9638 | 0.9643 | 0.9285 | 0.9330 | 0.9194 | 0.9182 | 0.8866 | 0.8927 | 0.7282 | 0.6615 | 0.6905 | 0.6907 |
| 2 | 0.9197 | 0.9288 | 0.9188 | 0.9246 | 0.9092 | 0.9205 | 0.9106 | 0.9168 | 0.9296 | 0.9695 | 0.9277 | 0.9435 |
| 4 | 0.9685 | 0.9680 | 0.9679 | 0.9675 | 0.9562 | 0.9574 | 0.9562 | 0.9568 | 0.9406 | 0.9406 | 0.9383 | 0.9456 |
| 6 | 0.9927 | 0.9926 | 0.9925 | 0.9926 | 0.9851 | 0.9852 | 0.9850 | 0.9852 | 0.9467 | 0.9466 | 0.9467 | 0.9490 |
| 12 | 1.0001 | 1.0000 | 1.0000 | 1.0001 | 1.0002 | 1.0005 | 1.0005 | 1.0005 | 0.9803 | 0.9820 | 0.9803 | 0.9802 |
| 18 | 0.9997 | 0.9996 | 0.9996 | 0.9996 | 1.0038 | 1.0037 | 1.0037 | 1.0037 | 0.9957 | 0.9947 | 0.9957 | 0.9957 |
| 24 | 1.0009 | 1.0008 | 1.0008 | 1.0008 | 1.0010 | 1.0009 | 1.0009 | 1.0009 | 0.9978 | 0.9977 | 0.9978 | 0.9978 |
| 36 | 0.9998 | 0.9997 | 0.9997 | 0.9997 | 0.9993 | 0.9995 | 0.9995 | 0.9995 | 0.9986 | 0.9986 | 0.9986 | 0.9986 |

# D. FORECASTING MACROECONOMIC AGGREGATES IN A SMALL OPEN ECONOMY: CANADA

Using the Canadian balanced monthly panel of Boivin et al. (2009b), we performed an out-of-sample forecasting exercise similar to the one described above for U.S. data. This panel comprises 332 time series from 1981 to 2008. The evaluation period is 1998-2008. All series were initially transformed to induce stationarity. For this panel, the time and cross-section dimensions are close. The number of factors and lag orders across the forecasting models are the same as for the U.S. data.

The results in Table 3 are quite similar to the U.S. ones: FAVARMA models improve the forecasts of key macroeconomic indicators across several horizons. In particular, VARMA factors produce the best forecasts of employment at all horizons (except for 12, 18 and 24 months). For CPI inflation, the Diffusion index model has the best performance at short horizons of 1 and 2 months, at the 18-month horizon, and for long horizons of 3 and 4 years. FAVARMA models in Final MA form outperform other approaches at 4, 6, 12 and 24 month horizons. Finally, ARMA models yield the smallest RMSE for PPI inflation at short horizons (1, 2, 4 and 6 months), while Diagonal MA and Final AR FAVARMA models have the best performance at horizons of 12 and 36 months respectively.

In Table 4, we present the MSE of all factor model predictions relative to ARMA forecasts. Boldface numbers highlight cases where the ARMA model outperforms the factor-based model in terms of MSE. For the employment growth rate, the ARMA model outperforms all three one-step models, except at the 2-month horizon. On the other hand, three FAVARMA forms and the Sequential FAVAR model do better than ARMA models for all horizons. When forecasting CPI inflation, the two FAVARMA MA forms appear preferable to ARMA models at all horizons. Finally, for PPI inflation, ARMA models exhibit the best performance at short horizons.

It is also of interest to see how FAVARMA forecasts fare in comparison with those of FAVAR models. In Table 5, we present MSE of FAVARMA forecasting models relative to direct and sequential FAVAR approaches. Numbers in bold character indicate cases where the FAVARMA specification performs better than the FAVAR one. The FAVARMA models dominate in most cases, especially the two MA FAVARMA specifications.

Table 3: RMSE relative to direct AR$(p)$ forecasts – Canadian data

| Employment growth rate | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Horizon | Unrestricted | DI | DI AR | Direct | Sequential | Diag MA | Diag AR | Final MA | Final AR | ARMA |
| 1 | 1.0221 | 1.0165 | 1.0920 | 0.9854 | 0.9854 | **0.9410** | 0.9854 | 0.9601 | 1.0362 | 1.0151 |
| 2 | 0.9874 | 0.9751 | 0.9457 | 0.9998 | 0.9920 | **0.9059** | 0.9920 | 0.9236 | 1.0597 | 1.0092 |
| 4 | 1.0604 | 1.0865 | 1.1204 | 0.9783 | 0.9399 | 0.9298 | 0.9399 | **0.9221** | 1.0503 | 1.0060 |
| 6 | 1.1928 | 1.1408 | 1.1667 | 1.1130 | 0.9760 | 0.9641 | 0.9760 | **0.9286** | 1.0615 | 1.0011 |
| 12 | **0.9822** | 1.1197 | 1.2073 | 1.0402 | 0.9914 | 1.0194 | 0.9914 | 0.9938 | 1.0889 | 1.0760 |
| 18 | 1.2135 | 1.5923 | 1.6208 | 1.3230 | **0.9792** | 1.0282 | 1.0740 | 0.9845 | 1.1923 | 1.1054 |
| 24 | 1.3133 | 1.9476 | 1.9595 | 1.1989 | **0.9803** | 1.0290 | 1.0022 | 0.9819 | 1.1401 | 1.0937 |
| 36 | 1.7336 | 2.1289 | 2.2198 | 1.5687 | 0.9201 | 0.9395 | 0.9441 | **0.9190** | 1.0639 | 1.0442 |
| 48 | 1.7698 | 1.5115 | 1.2833 | 1.7333 | 0.9788 | 0.9734 | 0.9905 | **0.9608** | 1.0926 | 1.0829 |

| Consumer price index growth rate: all items | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Horizon | Unrestricted | DI | DI AR | Direct | Sequential | Diag MA | Diag AR | Final MA | Final AR | ARMA |
| 1 | 0.8779 | **0.8501** | 0.8567 | 0.9146 | 0.9146 | 0.8563 | 0.9130 | 0.8647 | 0.9512 | 0.8811 |
| 2 | 0.9028 | **0.8720** | 0.8790 | 0.9946 | 0.9804 | 0.8895 | 0.9804 | 0.9040 | 0.9798 | 0.9226 |
| 4 | 0.9139 | 0.9082 | 0.9000 | 0.9737 | 0.9328 | 0.8826 | 0.9328 | **0.8816** | 0.9430 | 0.9069 |
| 6 | 0.8800 | 0.8701 | 0.8811 | 0.9307 | 0.8853 | 0.8403 | 0.8853 | **0.8399** | 0.8900 | 0.9062 |
| 12 | 0.9921 | 1.0585 | 1.0140 | 1.0178 | 0.9845 | 0.9318 | 0.9845 | **0.9070** | 1.0255 | 1.0207 |
| 18 | 1.0114 | 1.0143 | **1.0083** | 1.0362 | 1.0138 | 1.0504 | 1.0847 | 1.0130 | 1.0368 | 1.1184 |
| 24 | 0.9810 | 1.0563 | 1.0743 | 0.9671 | 0.9460 | 0.9655 | 0.9938 | **0.9508** | 1.0340 | 1.0804 |
| 36 | **0.9844** | 1.1165 | 1.1126 | 1.0140 | 1.0179 | 1.0325 | 1.0309 | 1.0160 | 1.1187 | 1.1287 |
| 48 | **0.9919** | 1.3307 | 1.3174 | 1.0908 | 1.0550 | 1.0415 | 1.0318 | 1.0554 | 1.1554 | 1.1832 |

| Producer price index growth rate: all manufacturing | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Horizon | Unrestricted | DI | DI AR | Direct | Sequential | Diag MA | Diag AR | Final MA | Final AR | ARMA |
| 1 | 1.0079 | 1.0035 | 1.0094 | 1.0097 | 1.0097 | 0.9985 | 1.0070 | 1.0175 | 1.0443 | **0.9931** |
| 2 | 1.0088 | 0.9732 | 0.9835 | 1.0317 | 1.0077 | 0.9852 | 1.0077 | 0.9874 | 1.0499 | **0.9729** |
| 4 | 0.9841 | 1.0255 | 1.0280 | 1.0115 | 0.9810 | 0.9986 | 0.9810 | 0.9852 | 1.0483 | **0.9803** |
| 6 | 0.9759 | 1.0083 | 1.0103 | 0.9885 | 0.9701 | 0.9830 | 0.9701 | 0.9781 | 0.9958 | **0.9580** |
| 12 | 1.0246 | 1.0274 | 1.0294 | 1.0183 | 1.0142 | **0.9916** | 1.0142 | 0.9942 | 0.9973 | 1.0123 |
| 18 | **0.9740** | 0.9998 | 1.0026 | 0.9905 | 0.9828 | 0.9789 | 0.9837 | 0.9815 | 0.9894 | 0.9842 |
| 24 | **0.9927** | 1.0204 | 1.0230 | 1.0159 | 1.0027 | 0.9956 | 1.0018 | 0.9981 | 0.9984 | 1.0040 |
| 36 | 1.0363 | 1.0763 | 1.0947 | 0.9850 | 0.9831 | 0.9790 | 0.9814 | 0.9804 | **0.9755** | 0.9842 |
| 48 | **0.9890** | 1.0761 | 1.0632 | 0.9927 | 1.0108 | 1.0032 | 1.0050 | 1.0110 | 0.9969 | 1.0143 |

Note – The numbers in bold character indicate which model yields the lowest forecast MSE.

Table 4: RMSE relative to ARMA$(p, q)$ forecasts – Canadian data

| Horizon | Unrestricted | DI | DI AR | Direct | Sequential | Diag MA | Diag AR | Final MA | Final AR |
|---|---|---|---|---|---|---|---|---|---|
| Employment growth rate | | | | | | | | | |
| 1 | **1.0069** | **1.0014** | **1.0758** | 0.9707 | 0.9707 | 0.9270 | 0.9707 | 0.9458 | **1.0208** |
| 2 | 0.9784 | 0.9662 | 0.9371 | 0.9907 | 0.9830 | 0.8976 | 0.9830 | 0.9152 | **1.0500** |
| 4 | **1.0541** | **1.0800** | **1.1137** | 0.9725 | 0.9343 | 0.9243 | 0.9343 | 0.9166 | **1.0440** |
| 6 | **1.1915** | **1.1395** | **1.1654** | **1.1118** | 0.9749 | 0.9630 | 0.9749 | 0.9276 | **1.0603** |
| 12 | 0.9128 | **1.0406** | **1.1220** | 0.9667 | 0.9214 | 0.9474 | 0.9214 | 0.9236 | **1.0120** |
| 18 | **1.0978** | **1.4405** | **1.4663** | **1.1969** | 0.8858 | 0.9302 | 0.9716 | 0.8906 | **1.0786** |
| 24 | **1.2008** | **1.7807** | **1.7916** | **1.0962** | 0.8963 | 0.9408 | 0.9163 | 0.8978 | **1.0424** |
| 36 | **1.6602** | **2.0388** | **2.1258** | **1.5023** | 0.8812 | 0.8997 | 0.9041 | 0.8801 | **1.0189** |
| 48 | **1.6343** | **1.3958** | **1.1851** | **1.6006** | 0.9039 | 0.8989 | 0.9147 | 0.8872 | **1.0090** |
| Consumer price index growth rate: all items | | | | | | | | | |
| Horizon | Unrestricted | DI | DI AR | Direct | Sequential | Diag MA | Diag AR | Final MA | Final AR |
| 1 | 0.9964 | 0.9648 | 0.9723 | **1.0380** | **1.0380** | 0.9719 | **1.0362** | 0.9814 | **1.0796** |
| 2 | 0.9785 | 0.9452 | 0.9527 | **1.0780** | **1.0626** | 0.9641 | **1.0626** | 0.9798 | **1.0620** |
| 4 | **1.0077** | **1.0014** | 0.9924 | **1.0737** | **1.0286** | 0.9732 | **1.0286** | 0.9721 | **1.0398** |
| 6 | 0.9711 | 0.9602 | 0.9723 | **1.0270** | 0.9769 | 0.9273 | 0.9769 | 0.9268 | 0.9821 |
| 12 | 0.9720 | **1.0370** | 0.9934 | 0.9972 | 0.9645 | 0.9129 | 0.9645 | 0.8886 | **1.0047** |
| 18 | 0.9043 | 0.9069 | 0.9016 | 0.9265 | 0.9065 | 0.9392 | 0.9699 | 0.9058 | 0.9270 |
| 24 | 0.9080 | 0.9777 | 0.9944 | 0.8951 | 0.8756 | 0.8937 | 0.9198 | 0.8800 | 0.9571 |
| 36 | 0.8722 | 0.9892 | 0.9857 | 0.8984 | 0.9018 | 0.9148 | 0.9134 | 0.9002 | 0.9911 |
| 48 | 0.8383 | **1.1247** | **1.1134** | 0.9219 | 0.8916 | 0.8802 | 0.8720 | 0.8920 | 0.9765 |
| Producer price index growth rate: all manufacturing | | | | | | | | | |
| Horizon | Unrestricted | DI | DI AR | Direct | Sequential | Diag MA | Diag AR | Final MA | Final AR |
| 1 | **1.0149** | **1.0105** | **1.0164** | **1.0167** | **1.0167** | **1.0054** | **1.0140** | **1.0246** | **1.0516** |
| 2 | **1.0369** | **1.0003** | **1.0109** | **1.0604** | **1.0358** | **1.0126** | **1.0358** | **1.0149** | **1.0791** |
| 4 | **1.0039** | **1.0461** | **1.0487** | **1.0318** | **1.0007** | **1.0187** | **1.0007** | **1.0050** | **1.0694** |
| 6 | **1.0187** | **1.0525** | **1.0546** | **1.0318** | **1.0126** | **1.0261** | **1.0126** | **1.0210** | **1.0395** |
| 12 | **1.0122** | **1.0149** | **1.0169** | **1.0059** | **1.0019** | 0.9796 | **1.0019** | 0.9821 | 0.9852 |
| 18 | 0.9896 | **1.0159** | **1.0187** | **1.0064** | 0.9986 | 0.9946 | 0.9995 | 0.9973 | **1.0053** |
| 24 | 0.9887 | **1.0163** | **1.0189** | **1.0119** | 0.9987 | 0.9916 | 0.9978 | 0.9941 | 0.9944 |
| 36 | **1.0529** | **1.0936** | **1.1123** | **1.0008** | 0.9989 | 0.9947 | 0.9972 | 0.9961 | 0.9912 |
| 48 | 0.9751 | **1.0609** | **1.0482** | 0.9787 | 0.9965 | 0.9891 | 0.9908 | 0.9967 | 0.9828 |

Note – The numbers in bold character present cases where the ARMA model outperforms the factor-based models in terms of MSE.

Table 5: MSE of FAVARMA relative to FAVAR forecasting models – Canadian data

| | Employment growth rate | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | VARMA/Direct | | | | VARMA/Sequential | | | |
| Horizon | Diag MA | Diag AR | Final MA | Final AR | Diag MA | Diag AR | Final MA | Final AR |
| 1 | **0.9500** | **0.9551** | **0.9671** | **0.9515** | **0.9500** | **0.9551** | **0.9671** | **0.9515** |
| 2 | **0.8934** | **0.9060** | **0.8971** | **0.8982** | **0.8508** | **0.8628** | **0.8544** | **0.8554** |
| 4 | **0.8353** | **0.9248** | **0.8375** | **0.8852** | **0.7325** | **0.8110** | **0.7344** | **0.7762** |
| 6 | **0.7875** | **0.8936** | **0.7836** | **0.8385** | **0.6836** | **0.7757** | **0.6802** | **0.7279** |
| 12 | **0.9154** | 1.1173 | **0.9215** | **0.9486** | **0.7315** | **0.8928** | **0.7364** | **0.7580** |
| 18 | **0.9858** | 1.4161 | **0.9956** | 1.0510 | **0.8403** | 1.2071 | **0.8487** | **0.8959** |
| 24 | **0.9733** | 1.7694 | **0.9833** | 1.0921 | **0.9075** | 1.6496 | **0.9168** | 1.0182 |
| 36 | **0.7971** | 2.4510 | **0.7993** | **0.9540** | **0.9457** | 2.9080 | **0.9484** | 1.1318 |
| 48 | **0.8968** | 5.0005 | **0.8969** | 1.0559 | **0.9581** | 5.3424 | **0.9582** | 1.1281 |

| | Consumer price index growth rate: all items | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | VARMA/Direct | | | | VARMA/Sequential | | | |
| Horizon | Diag MA | Diag AR | Final MA | Final AR | Diag MA | Diag AR | Final MA | Final AR |
| 1 | **0.9759** | **0.9845** | **0.9787** | **0.9763** | **0.9759** | **0.9845** | **0.9787** | **0.9763** |
| 2 | **0.9424** | **0.9291** | **0.9456** | **0.9341** | **0.9312** | **0.9180** | **0.9343** | **0.9229** |
| 4 | **0.9029** | **0.8817** | **0.9016** | **0.8811** | **0.8732** | **0.8527** | **0.8720** | **0.8521** |
| 6 | **0.9023** | **0.8891** | **0.8994** | **0.8825** | **0.8370** | **0.8248** | **0.8343** | **0.8186** |
| 12 | **0.9587** | **0.9778** | **0.9573** | **0.9429** | **0.8339** | **0.8505** | **0.8327** | **0.8201** |
| 18 | 1.0347 | 1.1696 | 1.0371 | 1.0674 | **0.8843** | **0.9996** | **0.8863** | **0.9122** |
| 24 | 1.0532 | 1.3283 | 1.0556 | 1.1260 | **0.9229** | 1.1640 | **0.9250** | **0.9867** |
| 36 | **0.9540** | 1.6253 | **0.9512** | 1.0622 | **0.9571** | 1.6305 | **0.9542** | 1.0655 |
| 48 | 1.0079 | 2.5086 | 1.0050 | 1.0946 | **0.9663** | 2.4048 | **0.9635** | 1.0494 |

| | Producer price index growth rate: all manufacturing | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | VARMA/Direct | | | | VARMA/Sequential | | | |
| Horizon | Diag MA | Diag AR | Final MA | Final AR | Diag MA | Diag AR | Final MA | Final AR |
| 1 | **0.9909** | 1.0221 | **0.9597** | **0.9831** | **0.9909** | 1.0221 | **0.9597** | **0.9831** |
| 2 | **0.9685** | **0.9808** | **0.9648** | **0.9540** | **0.9567** | **0.9689** | **0.9531** | **0.9424** |
| 4 | **0.9624** | 1.0257 | **0.9516** | **0.9417** | **0.9840** | 1.0487 | **0.9730** | **0.9628** |
| 6 | **0.9611** | 1.0389 | **0.9604** | **0.9593** | **0.9847** | 1.0644 | **0.9840** | **0.9829** |
| 12 | **0.8846** | 1.1051 | **0.8828** | **0.9618** | 1.0062 | 1.2570 | 1.0042 | 1.0941 |
| 18 | **0.8527** | 1.0567 | **0.8542** | **0.9274** | **0.9213** | 1.1416 | **0.9228** | 1.0020 |
| 24 | **0.9273** | 1.2462 | **0.9244** | 1.0213 | **0.8837** | 1.1875 | **0.8809** | **0.9732** |
| 36 | **0.8439** | 1.4783 | **0.8495** | **0.9246** | **0.8795** | 1.5408 | **0.8854** | **0.9637** |
| 48 | **0.8858** | 2.4185 | **0.8904** | **0.9469** | **0.8695** | 2.3742 | **0.8741** | **0.9295** |

Note – The numbers in bold character indicate cases where the FAVARMA model performs better than the FAVAR model.

## E. DATA

The data used in our empirical application are presented in this appendix. US data are taken from Boivin, Giannoni and Stevanovic (2009a), while the Canadian data are from Boivin, Giannoni and Stevanovic (2009b). The transformation codes (labeled T-Code) are: 1 - no transformation; 2 - first difference; 4 - logarithm; 5 - first difference of logarithm.

# US Data

| No. | Series Code | T-Code | Series Description |
|---|---|---|---|
| | | | **Real output and income** |
| **1** | IPS10 | 5 | INDUSTRIAL PRODUCTION INDEX - TOTAL INDEX |
| **2** | IPS11 | 5 | INDUSTRIAL PRODUCTION INDEX - PRODUCTS, TOTAL |
| **3** | IPS12 | 5 | INDUSTRIAL PRODUCTION INDEX - CONSUMER GOODS |
| **4** | IPS13 | 5 | INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS |
| **5** | IPS14 | 5 | INDUSTRIAL PRODUCTION INDEX - AUTOMOTIVE PRODUCTS |
| **6** | IPS18 | 5 | INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS |
| **7** | IPS25 | 5 | INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT |
| **8** | IPS29 | 5 | INDUSTRIAL PRODUCTION INDEX - DEFENSE AND SPACE EQUIPMENT |
| **9** | IPS299 | 5 | INDUSTRIAL PRODUCTION INDEX - FINAL PRODUCTS |
| **10** | IPS306 | 5 | INDUSTRIAL PRODUCTION INDEX - FUELS |
| **11** | IPS32 | 5 | INDUSTRIAL PRODUCTION INDEX - MATERIALS |
| **12** | IPS34 | 5 | INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS |
| **13** | IPS38 | 5 | INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS |
| **14** | IPS43 | 5 | INDUSTRIAL PRODUCTION INDEX - MANUFACTURING (SIC) |
| **15** | PMP | 1 | NAPM PRODUCTION INDEX (PERCENT) |
| **16** | PMI | 1 | PURCHASING MANAGERS' INDEX (SA) |
| **17** | UTL11 | 1 | CAPACITY UTILIZATION - MANUFACTURING (SIC) |
| **18** | YPR | 5 | PERS INCOME CH 2000 $,SA-US |
| **19** | YPDR | 5 | DISP PERS INCOME,BILLIONS OF CH (2000) $,SAAR-US |
| **20** | YP@V00C | 5 | PERS INCOME LESS TRSF PMT CH 2000 $,SA-US |
| **21** | SAVPER | 2 | PERS SAVING,BILLIONS OF $,SAAR-US |
| **22** | SAVPRATE | 1 | PERS SAVING AS PERCENTAGE OF DISP PERS INCOME,PERCENT,SAAR-US |
| | | | **Employment and hours** |
| **23** | LHEL | 5 | INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA) |
| **24** | LHELX | 4 | EMPLOYMENT: RATIO; HELP-WANTED ADS:NO. UNEMPLOYED CLF |
| **25** | LHEM | 5 | CIVILIAN LABOR FORCE: EMPLOYED, TOTAL (THOUS.,SA) |
| **26** | LHNAG | 5 | CIVILIAN LABOR FORCE: EMPLOYED, NONAGRIC.INDUSTRIES (THOUS.,SA) |
| **27** | LHTUR | 1 | UNEMPLOYMENT RATE: BOTH SEXES, 16-19 YEARS (%,SA) |
| **28** | LHU14 | 1 | UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA) |
| **29** | LHU15 | 1 | UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA) |
| **30** | LHU26 | 1 | UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA) |
| **31** | LHU27 | 1 | UNEMPLOY.BY DURATION: PERSONS UNEMPL.27 WKS + (THOUS,SA) |
| **32** | LHU5 | 1 | UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA) |
| **33** | LHU680 | 1 | UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA) |
| **34** | LHUEM | 5 | CIVILIAN LABOR FORCE: UNEMPLOYED, TOTAL (THOUS.,SA) |
| **35** | AHPCON | 1 | AVG HR EARNINGS OF PROD WKRS: CONSTRUCTION ($,SA) |
| **36** | AHPMF | 1 | AVG HR EARNINGS OF PROD WKRS: MANUFACTURING ($,SA) |
| **37** | PMEMP | 1 | NAPM EMPLOYMENT INDEX (PERCENT) |
| **38** | CES002 | 5 | EMPLOYEES ON NONFARM PAYROLLS - TOTAL PRIVATE |
| **39** | CES003 | 5 | EMPLOYEES ON NONFARM PAYROLLS - GOODS-PRODUCING |
| **40** | CES004 | 5 | EMPLOYEES ON NONFARM PAYROLLS - NATURAL RESOURCES AND MINING |
| **41** | CES011 | 5 | EMPLOYEES ON NONFARM PAYROLLS - CONSTRUCTION |
| **42** | CES015 | 5 | EMPLOYEES ON NONFARM PAYROLLS - MANUFACTURING |
| **43** | CES017 | 5 | EMPLOYEES ON NONFARM PAYROLLS - DURABLE GOODS |
| **44** | CES033 | 5 | EMPLOYEES ON NONFARM PAYROLLS - NONDURABLE GOODS |
| **45** | CES046 | 5 | EMPLOYEES ON NONFARM PAYROLLS - SERVICE-PROVIDING |
| **46** | CES048 | 5 | EMPLOYEES ON NONFARM PAYROLLS - TRADE, TRANSPORTATION, AND UTILITIES |
| **47** | CES049 | 5 | EMPLOYEES ON NONFARM PAYROLLS - WHOLESALE TRADE |
| **48** | CES053 | 5 | EMPLOYEES ON NONFARM PAYROLLS - RETAIL TRADE |
| **49** | CES088 | 5 | EMPLOYEES ON NONFARM PAYROLLS - FINANCIAL ACTIVITIES |
| **50** | CES140 | 5 | EMPLOYEES ON NONFARM PAYROLLS - GOVERNMENT |
| **51** | CES151 | 1 | AVG WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - GOODS-PRODUCING |
| **52** | CES153 | 1 | AVG WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - CONSTRUCTION |
| **53** | CES154 | 1 | AVG WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - MANUFACTURING |
| **54** | CES155 | 1 | AVG WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - MANUFACT. OVERTIME HOURS |
| **55** | CES156 | 1 | AVG WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - DURABLE GOODS |
| **56** | CES275 | 5 | AVG HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - GOODS-PRODUCING |
| **57** | CES277 | 5 | AVG HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - CONSTRUCTION |
| **58** | CES278 | 5 | AVG HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - MANUFACTURING |
| | | | **Real Consumption** |
| **59** | JQCR | 5 | REAL PERSONAL CONS EXP QUANTITY INDEX (200=100), SAAR |
| **60** | JQCNR | 5 | REAL PERSONAL CONS EXP-NONDURABLE GOODS QUANTITY INDEX (200=100), SAAR |
| **61** | JQCDR | 5 | REAL PERSONAL CONS EXP-DURABLE GOODS QUANTITY INDEX (200=100), SAAR |
| **62** | JQCSVR | 5 | REAL PERSONAL CONS EXP-SERVICES QUANTITY INDEX (200=100), SAAR |

| | | | |
|---|---|---|---|
| | | | **Real inventories and orders** |
| **63** | MOCMQ | 5 | NEW ORDERS (NET) - CONSUMER GOODS and MATERIALS, 1996 DOLLARS (BCI) |
| **64** | MSONDQ | 5 | NEW ORDERS, NONDEFENSE CAPITAL GOODS, IN 1996 DOLLARS (BCI) |
| **65** | PMDEL | 1 | NAPM VENDOR DELIVERIES INDEX (PERCENT) |
| **66** | PMNO | 1 | NAPM NEW ORDERS INDEX (PERCENT) |
| **67** | PMNV | 1 | NAPM INVENTORIES INDEX (PERCENT) |
| | | | **Housing starts** |
| **68** | XMTOSA | 4 | RESIDENTIAL CONSTRUCTION PRIVATE HOUSING UNITS STARTED: TOTAL UNITS (THOUS.,SAAR) |
| **69** | HUSTSZ | 4 | HOUSING STARTS: TOTAL NEW PRIV HOUSING UNITS (THOUS.,SAAR) |
| **70** | HSFR | 4 | HOUSING STARTS:NONFARM(1947-58);TOTAL FARM&NONFARM(1959-)(THOUS.,SA |
| **71** | HSMW | 4 | HOUSING STARTS:MIDWEST(THOUS.U.)S.A. |
| **72** | HSNE | 4 | HOUSING STARTS:NORTHEAST (THOUS.U.)S.A. |
| **73** | HSSOU | 4 | HOUSING STARTS:SOUTH (THOUS.U.)S.A. |
| **74** | HSWST | 4 | HOUSING STARTS:WEST (THOUS.U.)S.A. |
| | | | **Exchange rates** |
| **75** | EXRCAN | 5 | FOREIGN EXCHANGE RATE: CANADA (CANADIAN $ PER U.S.$) |
| **76** | EXRUK | 5 | FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND) |
| **77** | EXRUS | 5 | UNITED STATES;EFFECTIVE EXCHANGE RATE(MERM)(INDEX NO.) |
| | | | **Price indexes** |
| **78** | PMCP | 1 | NAPM COMMODITY PRICES INDEX (PERCENT) |
| **79** | PW561 | 5 | PRODUCER PRICE INDEX: CRUDE PETROLEUM (82=100,NSA) |
| **80** | PWCMSA | 5 | PRODUCER PRICE INDEX:CRUDE MATERIALS (82=100,SA) |
| **81** | PWFCSA | 5 | PRODUCER PRICE INDEX:FINISHED CONSUMER GOODS (82=100,SA) |
| **82** | PWFSA | 5 | PRODUCER PRICE INDEX: FINISHED GOODS (82=100,SA) |
| **83** | PWIMSA | 5 | PRODUCER PRICE INDEX:INTERMED MAT.SUPPLIES & COMPONENTS(82=100,SA) |
| **84** | PUNEW | 5 | CPI-U: ALL ITEMS (82-84=100,SA) |
| **85** | PUS | 5 | CPI-U: SERVICES (82-84=100,SA) |
| **86** | PUXF | 5 | CPI-U: ALL ITEMS LESS FOOD (82-84=100,SA) |
| **87** | PUXHS | 5 | CPI-U: ALL ITEMS LESS SHELTER (82-84=100,SA) |
| **88** | PUXM | 5 | CPI-U: ALL ITEMS LESS MIDICAL CARE (82-84=100,SA) |
| **89** | PUXX | 5 | CPI-U: ALL ITEMS LESS FOOD AND ENERGY (82-84=100,SA) |
| **90** | PUC | 5 | CPI-U: COMMODITIES (82-84=100,SA) |
| **91** | PUCD | 5 | CPI-U: DURABLES (82-84=100,SA) |
| **92** | PU83 | 5 | CPI-U: APPAREL & UPKEEP (82-84=100,SA) |
| **93** | PU84 | 5 | CPI-U: TRANSPORTATION (82-84=100,SA) |
| **94** | PU85 | 5 | CPI-U: MEDICAL CARE (82-84=100,SA) |
| | | | **Stock prices** |
| **95** | FSDJ | 5 | COMMON STOCK PRICES: DOW JONES INDUSTRIAL AVERAGE |
| **96** | FSDXP | 1 | S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM) |
| **97** | FSPCOM | 5 | S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10) |
| **98** | FSPIN | 5 | S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10) |
| **99** | FSPXE | 1 | S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (%,NSA) |
| | | | **Money and credit quantity aggregates** |
| **100** | FM1 | 5 | MONEY STOCK: M1(CURR,TRAV.CKS,DEM DEP,OTHER CK'ABLE DEP)(BIL$,SA) |
| **101** | FM2 | 5 | MONEY STOCK:M2(M1+O'NITE RPS,EURO$,G/P&B/D MMMFS&SAV&SM TIME DEP(BIL$, |
| **102** | FMFBA | 5 | MONETARY BASE, ADJ FOR RESERVE REQUIREMENT CHANGES(MIL$,SA) |
| **103** | FMRNBA | 2 | DEPOSITORY INST RESERVES:NONBORROWED,ADJ RES REQ CHGS(MIL$,SA) |
| **104** | FMRRA | 5 | DEPOSITORY INST RESERVES:TOTAL,ADJ FOR RESERVE REQ CHGS(MIL$,SA) |
| **105** | CCINRV | 5 | CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19) |
| | | | **Miscellaneous** |
| **106** | UOMO83 | 1 | COMPOSITE INDEXES LEADING INDEX COMPONENT INDEX OF CONSUMER EXPECTATIONS UNITS: 1966.1=100 NSA, CONFBOARD AND U.MICH. |
| | | | **Interest rates and bonds** |
| **107** | FYGM3 | 1 | INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NSA) |
| **108** | FYGM6 | 1 | INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,6-MO.(% PER ANN,NSA) |
| **109** | FYGT1 | 1 | INTEREST RATE: U.S.TREASURY CONST MATURITIES,1-YR.(% PER ANN,NSA) |
| **110** | FYGT10 | 1 | INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA) |
| **111** | FYGT20 | 1 | INTEREST RATE: U.S.TREASURY CONST MATURITIES,20-YR.(% PER ANN,NSA) |
| **112** | FYGT3 | 1 | INTEREST RATE: U.S.TREASURY CONST MATURITIES,3-YR.(% PER ANN,NSA) |
| **113** | FYGT5 | 1 | INTEREST RATE: U.S.TREASURY CONST MATURITIES,5-YR.(% PER ANN,NSA) |
| **114** | FYPR | 1 | PRIME RATE CHG BY BANKS ON SHORT-TERM BUSINESS LOANS(% PER ANN,NSA) |
| **115** | FYAAAC | 1 | BOND YIELD: MOODY'S AAA CORPORATE (% PER ANNUM) |
| **116** | FYAAAM | 1 | BOND YIELD: MOODY'S AAA MUNICIPAL (% PER ANNUM) |
| **117** | FYAC | 1 | BOND YIELD: MOODY'S A CORPORATE (% PER ANNUM,NSA) |
| **118** | FYAVG | 1 | BOND YIELD: MOODY'S AVERAGE CORPORATE (% PER ANNUM) |
| **119** | FYBAAC | 1 | BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM) |
| **120** | SFYGM3 | 1 | FYGM3-FYFF |
| **121** | SFYGM6 | 1 | FYGM6-FYFF |
| **122** | SFYGT1 | 1 | FYGT1-FYFF |
| **123** | SFYGT5 | 1 | FYGT5-FYFF |
| **124** | SFYGT10 | 1 | FYGT10-FYFF |
| **125** | SFYAAAC | 1 | FYAAAC-FYFF |
| **126** | SFYBAAC | 1 | FYBAAC-FYFF |
| **127** | FYFF | 1 | INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (% PER ANNUM,NSA) |
| **128** | Bspread10Y | 1 | FYBAAC-FYGT10 |

# Canadian Data

| No. | StatCan no | Code | Series category |
|---|---|---|---|
| | | | **Table 326-0020 Consumer Price Index Canada, Provinces** |
| **1** | v41690973 | **5** | All-items (2002=100) |
| **2** | v41690974 | **5** | Food (2002=100) |
| **3** | v41690993 | **5** | Dairy products (2002=100) |
| **4** | v41691046 | **5** | Food purchased from restaurants (2002=100) |
| **5** | v41691051 | **5** | Rented accommodation (2002=100) |
| **6** | v41691055 | **5** | Owned accommodation (2002=100) |
| **7** | v41691065 | **5** | Natural gas (2002=100) |
| **8** | v41691066 | **5** | Fuel oil and other fuels (2002=100) |
| **9** | v41691108 | **5** | Clothing and footwear (2002=100) |
| **10** | v41691129 | **5** | Private transportation (2002=100) |
| **11** | v41691153 | **5** | Health and personal care (2002=100) |
| **12** | v41691170 | **5** | Recreation, education and reading (2002=100) |
| **13** | v41692942 | **5** | All-items excluding eight of the most volatile components (Bank of Canada definition) (2002=100) |
| **14** | v41691232 | **5** | All-items excluding food (2002=100) |
| **15** | v41691233 | **5** | All-items excluding food and energy (2002=100) |
| **16** | v41691238 | **5** | All-items excluding energy (2002=100) |
| **17** | v41691237 | **5** | Food and energy (2002=100) |
| **18** | v41691239 | **5** | Energy (2002=100) |
| **19** | v41691219 | **5** | Housing (1986 definition) (2002=100) |
| **20** | v41691222 | **5** | Goods (2002=100) |
| **21** | v41691223 | **5** | Durable goods (2002=100) |
| **22** | v41691225 | **5** | Non-durable goods (2002=100) |
| **23** | v41691229 | **5** | Goods excluding food purchased from stores and energy (2002=100) |
| **24** | v41691230 | **5** | Services (2002=100) |
| **25** | v41691231 | **5** | Services excluding shelter services (2002=100) |
| **26** | v41691244 | **5** | Newfoundland and Labrador; All-items (2002=100) |
| **27** | v41691369 | **5** | Newfoundland and Labrador; All-items excluding food and energy (2002=100) |
| **28** | v41691363 | **5** | Newfoundland and Labrador; Goods (2002=100) |
| **29** | v41691367 | **5** | Newfoundland and Labrador; Services (2002=100) |
| **30** | v41691379 | **5** | Prince Edward Island; All-items (2002=100) |
| **31** | v41691503 | **5** | Prince Edward Island; All-items excluding food and energy (2002=100) |
| **32** | v41691497 | **5** | Prince Edward Island; Goods (2002=100) |
| **33** | v41691501 | **5** | Prince Edward Island; Services (2002=100) |
| **34** | v41691513 | **5** | Nova Scotia; All-items (2002=100) |
| **35** | v41691638 | **5** | Nova Scotia; All-items excluding food and energy (2002=100) |
| **36** | v41691632 | **5** | Nova Scotia; Goods (2002=100) |
| **37** | v41691636 | **5** | Nova Scotia; Services (2002=100) |
| **38** | v41691648 | **5** | New Brunswick; All-items (2002=100) |
| **39** | v41691773 | **5** | New Brunswick; All-items excluding food and energy (2002=100) |
| **40** | v41691767 | **5** | New Brunswick; Goods (2002=100) |
| **41** | v41691771 | **5** | New Brunswick; Services (2002=100) |
| **42** | v41691783 | **5** | Quebec; All-items (2002=100) |
| **43** | v41691909 | **5** | Quebec; All-items excluding food and energy (2002=100) |
| **44** | v41691903 | **5** | Quebec; Goods (2002=100) |
| **45** | v41691907 | **5** | Quebec; Services (2002=100) |
| **46** | v41691919 | **5** | Ontario; All-items (2002=100) |
| **47** | v41692045 | **5** | Ontario; All-items excluding food and energy (2002=100) |
| **48** | v41692039 | **5** | Ontario; Goods (2002=100) |
| **49** | v41692043 | **5** | Ontario; Services (2002=100) |
| **50** | v41692055 | **5** | Manitoba; All-items (2002=100) |
| **51** | v41692181 | **5** | Manitoba; All-items excluding food and energy (2002=100) |
| **52** | v41692175 | **5** | Manitoba; Goods (2002=100) |
| **53** | v41692179 | **5** | Manitoba; Services (2002=100) |
| **54** | v41692191 | **5** | Saskatchewan; All-items (2002=100) |
| **55** | v41692317 | **5** | Saskatchewan; All-items excluding food and energy (2002=100) |
| **56** | v41692311 | **5** | Saskatchewan; Goods (2002=100) |
| **57** | v41692315 | **5** | Saskatchewan; Services (2002=100) |
| **58** | v41692327 | **5** | Alberta; All-items (2002=100) |
| **59** | v41692452 | **5** | Alberta; All-items excluding food and energy (2002=100) |
| **60** | v41692446 | **5** | Alberta; Goods (2002=100) |
| **61** | v41692450 | **5** | Alberta; Services (2002=100) |
| **62** | v41692462 | **5** | British Columbia; All-items (2002=100) |
| **63** | v41692588 | **5** | British Columbia; All-items excluding food and energy (2002=100) |
| **64** | v41692582 | **5** | British Columbia; Goods (2002=100) |
| **65** | v41692586 | **5** | British Columbia; Services (2002=100) |
| | | | **Table 026-0001 Building permits, residential values and number of units** |
| **66** | v14098 | **1** | Canada; Total dwellings (number of units) [D848383] |
| **67** | v41651 | **1** | Canada; Total dwellings (dollars - thousands) [D845521] |
| **68** | v13824 | **1** | Newfoundland and Labrador; Total dwellings (number of units) [D847651] |
| **69** | v41560 | **1** | Newfoundland and Labrador; Total dwellings (dollars - thousands) [D845363] |
| **70** | v13859 | **1** | Prince Edward Island; Total dwellings (number of units) [D847658] |
| **71** | v41595 | **1** | Prince Edward Island; Total dwellings (dollars - thousands) [D845370] |
| **72** | v13866 | **1** | Nova Scotia; Total dwellings (number of units) [D847665] |
| **73** | v41602 | **1** | Nova Scotia; Total dwellings (dollars - thousands) [D845377] |
| **74** | v13873 | **1** | New Brunswick; Total dwellings (number of units) [D847672] |
| **75** | v41609 | **1** | New Brunswick; Total dwellings (dollars - thousands) [D845384] |
| **76** | v13880 | **1** | Quebec; Total dwellings (number of units) [D847679] |
| **77** | v41616 | **1** | Quebec; Total dwellings (dollars - thousands) [D845391] |
| **78** | v13887 | **1** | Ontario; Total dwellings (number of units) [D847686] |
| **79** | v41623 | **1** | Ontario; Total dwellings (dollars - thousands) [D845398] |
| **80** | v13894 | **1** | Manitoba; Total dwellings (number of units) [D847693] |

| | | | |
|---|---|---|---|
| **81** | v41630 | **1** | Manitoba; Total dwellings (dollars - thousands) [D845405] |
| **82** | v13901 | **1** | Saskatchewan; Total dwellings (number of units) [D847700] |
| **83** | v41637 | **1** | Saskatchewan; Total dwellings (dollars - thousands) [D845412] |
| **84** | v13908 | **1** | Alberta; Total dwellings (number of units) [D847707] |
| **85** | v41644 | **1** | Alberta; Total dwellings (dollars - thousands) [D845419] |
| **86** | v13831 | **1** | British Columbia; Total dwellings (number of units) [D847714] |
| **87** | v41567 | **1** | British Columbia; Total dwellings (dollars - thousands) [D845426] |
| | | | **Table 027-0002 CMHC, housing starts, under constr and completions, SA** |
| **88** | v730040 | **1** | Canada; Total units (units - thousands) [J9001] |
| **89** | v729972 | **1** | Newfoundland and Labrador; Total units (units - thousands) [J7002] |
| **90** | v729973 | **1** | Prince Edward Island; Total units (units - thousands) [J7003] |
| **91** | v729974 | **1** | Nova Scotia; Total units (units - thousands) [J7004] |
| **92** | v729975 | **1** | New Brunswick; Total units (units - thousands) [J7005] |
| **93** | v729976 | **1** | Quebec; Total units (units - thousands) [J7006] |
| **94** | v729981 | **1** | Ontario; Total units (units - thousands) [J7008] |
| **95** | v729987 | **1** | Manitoba; Total units (units - thousands) [J7011] |
| **96** | v729988 | **1** | Saskatchewan; Total units (units - thousands) [J7012] |
| **97** | v729989 | **1** | Alberta; Total units (units - thousands) [J7013] |
| **98** | v729990 | **1** | British Columbia; Total units (units - thousands) [J7014] |
| | | | **Table 377-0003 Business leading indicators for Canada** |
| **99** | v7677 | **1** | Average work week, manufacturing; Smoothed (hours) [D100042] |
| **100** | v7680 | **1** | Housing index; Smoothed (index, 1992=100) [D100043] |
| **101** | v7681 | **5** | United States composite leading index; Smoothed (index, 1992=100) [D100044] |
| **102** | v7682 | **5** | Money supply; Smoothed (dollars, 1992 - millions) [D100045] |
| **103** | v7683 | **5** | New orders, durable goods; Smoothed (dollars, 1992 - millions) [D100046] |
| **104** | v7684 | **5** | Retail trade, furniture and appliances; Smoothed (dollars, 1992 - millions) [D100047] |
| **105** | v7686 | **1** | Shipment to inventory ratio, finished products; Smoothed (ratio) [D100049] |
| **106** | v7678 | **5** | Stock price index, TSE 300; Smoothed (index, 1975=1000) [D100050] |
| **107** | v7679 | **5** | Business and personal services employment; Smoothed (persons - thousands) [D100051] |
| **108** | v7688 | **5** | Composite index of 10 indicators; Smoothed (index, 1992=100) [D100053] |
| | | | **Table 379-0027 GDP at basic prices, by NAICS, Canada, SA, 2002 constant prices** |
| **109** | v41881478 | **5** | All industries [T001] (dollars - millions) |
| **110** | v41881480 | **5** | Business sector, goods [T003] (dollars - millions) |
| **111** | v41881481 | **5** | Business sector, services [T004] (dollars - millions) |
| **112** | v41881482 | **5** | Non-business sector industries [T005] (dollars - millions) |
| **113** | v41881485 | **5** | Goods-producing industries [T008] (dollars - millions) |
| **114** | v41881486 | **5** | Service-producing industries [T009] (dollars - millions) |
| **115** | v41881487 | **5** | Industrial production [T010] (dollars - millions) |
| **116** | v41881488 | **5** | Non-durable manufacturing industries [T011] (dollars - millions) |
| **117** | v41881489 | **5** | Durable manufacturing industries [T012] (dollars - millions) |
| **118** | v41881494 | **5** | Agriculture, forestry, fishing and hunting [11] (dollars - millions) |
| **119** | v41881501 | **5** | Mining and oil and gas extraction [21] (dollars - millions) |
| **120** | v41881524 | **5** | Residential building construction [230A] (dollars - millions) |
| **121** | v41881525 | **5** | Non-residential building construction [230B] (dollars - millions) |
| **122** | v41881527 | **5** | Manufacturing [31-33] (dollars - millions) |
| **123** | v41881555 | **5** | Wood product manufacturing [321] (dollars - millions) |
| **124** | v41881564 | **5** | Paper manufacturing [322] (dollars - millions) |
| **125** | v41881602 | **5** | Rubber product manufacturing [3262] (dollars - millions) |
| **126** | v41881606 | **5** | Non-metallic mineral product manufacturing [327] (dollars - millions) |
| **127** | v41881637 | **5** | Machinery manufacturing [333] (dollars - millions) |
| **128** | v41881654 | **5** | Electrical equipment, appliance and component manufacturing [335] (dollars - millions) |
| **129** | v41881662 | **5** | Transportation equipment manufacturing [336] (dollars - millions) |
| **130** | v41881663 | **5** | Motor vehicle manufacturing [3361] (dollars - millions) |
| **131** | v41881674 | **5** | Aerospace product and parts manufacturing [3364] (dollars - millions) |
| **132** | v41881675 | **5** | Railroad rolling stock manufacturing [3365] (dollars - millions) |
| **133** | v41881688 | **5** | Wholesale trade [41] (dollars - millions) |
| **134** | v41881689 | **5** | Retail trade [44-45] (dollars - millions) |
| **135** | v41881690 | **5** | Transportation and warehousing [48-49] (dollars - millions) |
| **136** | v41881699 | **5** | Pipeline transportation [486] (dollars - millions) |
| **137** | v41881724 | **5** | Finance, insurance, realÂ estate, rental and leasing and management of companies and enterprises [5A] (dollars - millions) |
| **138** | v41881756 | **5** | Educational services [61] (dollars - millions) |
| **139** | v41881759 | **5** | Health care and social assistance [62] (dollars - millions) |
| **140** | v41881776 | **5** | Federal government public administration [911] (dollars - millions) |
| **141** | v41881777 | **5** | Defence services [9111] (dollars - millions) |
| **142** | v41881779 | **5** | Provincial and territorial public administration [912] (dollars - millions) |
| **143** | v41881780 | **5** | Local, municipal and regional public administration [913] (dollars - millions) |
| | | | **Tables 329-00(46,38,39) Industrial price indexes, 1997=100** |
| **144** | v1575728 | **5** | Transformer equipment (index, 1997=100) [P5648] |
| **145** | v1575754 | **5** | Electric motors and generators (index, 1997=100) [P5674] |
| **146** | v1575886 | **5** | Diesel fuel (index, 1997=100) [P5806] |
| **147** | v1575925 | **5** | Light fuel oil (index, 1997=100) [P5845] |
| **148** | v1575903 | **5** | Heavy fuel oil (index, 1997=100) [P5823] |
| **149** | v1575934 | **5** | Lubricating oils and greases (index, 1997=100) [P5854] |
| **150** | v1575958 | **5** | Asphalt mixtures and emulsions (index, 1997=100) [P5878] |
| **151** | v1575457 | **5** | Industrial trucks, tractors and parts (index, 1997=100) [P5329] |
| **152** | v1575493 | **5** | Parts, air conditioning and refrigeration equipment (index, 1997=100) [P5365] |
| **153** | v1575511 | **5** | Food products industrial machinery and equipment (index, 1997=100) [P5383] |
| **154** | v1575557 | **5** | Trucks, chassis, tractors, commercial (index, 1997=100) [P5429] |
| **155** | v1575610 | **5** | Motor vehicle engine parts (index, 1997=100) [P5482] |
| **156** | v3860051 | **5** | Motor vehicle brakes (index, 1997=100) [P5512] |
| **157** | v3822562 | **5** | All manufacturing (index, 1997=100) [P6253] |
| **158** | v3825177 | **5** | Total excluding food and beverage manufacturing (index, 1997=100) [P6491] |

| | | | |
|---|---|---|---|
| **159** | v3825178 | **5** | Food and beverage manufacturing [311, 3121] (index, 1997=100) [P6492] |
| **160** | v3825179 | **5** | Food and beverage manufacturing excluding alcoholic beverages (index, 1997=100) [P6493] |
| **161** | v3825180 | **5** | Non-food (including alcoholic beverages) manufacturing (index, 1997=100) [P6494] |
| **162** | v3825181 | **5** | Basic manufacturing industries [321, 322, 327, 331] (index, 1997=100) [P6495] |
| **163** | v3825183 | **5** | Primary metal manufacturing excluding precious metals (index, 1997=100) [P6497] |
| | | | **Table 176-0001 Commodity price index, US$ (index, 82-90=100)** |
| **164** | v36382 | **5** | Total, all commodities (index, 82-90=100) [B3300] |
| **165** | v36383 | **5** | Total excluding energy (index, 82-90=100) [B3301] |
| **166** | v36384 | **5** | Energy (index, 82-90=100) [B3302] |
| **167** | v36385 | **5** | Food (index, 82-90=100) [B3303] |
| **168** | v36386 | **5** | Industrial materials (index, 82-90=100) [B3304] |
| | | | **Tables 176-00(46,47), 184-0002 Stock market statistics** |
| **169** | v37412 | **5** | Toronto Stock Exchange, value of shares traded (dollars - millions) [B4213] |
| **170** | v37413 | **5** | Toronto Stock Exchange, volume of shares traded (shares - millions) [B4214] |
| **171** | v37414 | **5** | United States common stocks, Dow-Jones industrials, high (index) [B4218] |
| **172** | v37415 | **5** | United States common stocks, Dow-Jones industrials, low (index) [B4219] |
| **173** | v37416 | **5** | United States common stocks, Dow-Jones industrials, close (index) [B4220] |
| **174** | v37419 | **5** | New York Stock Exchange, customers' debit balances (dollars - millions) [B4223] |
| **175** | v37420 | **5** | New York Stock Exchange, customers' free credit balance (dollars - millions) [B4224] |
| **176** | v122620 | **5** | Standard and Poor's/Toronto Stock Exchange Composite Index, close (index, 1975=1000) [B4237] |
| **177** | v122628 | **1** | Toronto Stock Exchange, stock dividend yields (composite), closing quotations (percent) [B4245] |
| **178** | v122629 | **1** | Toronto Stock Exchange, price earnings ratio, closing quotations (ratio) [B4246] |
| **179** | v6384 | **5** | Total volume; Value of shares traded (dollars - millions) [D4560] |
| **180** | v6385 | **5** | Industrials; Value of shares traded (dollars - millions) [D4558] |
| **181** | v6386 | **5** | Mining and oils; Value of shares traded (dollars - millions) [D4559] |
| | | | **Table 176-0064 Foreign exchange rates** |
| **183** | v37426 | **1** | United States dollar, noon spot rate, average (dollars) [B3400] |
| **184** | v37437 | **1** | United States dollar, 90-day forward noon rate (dollars) [B3401] |
| **185** | v37452 | **1** | Danish krone, noon spot rate, average (dollars) [B3403] |
| **186** | v37456 | **1** | Japanese yen, noon spot rate, average (dollars) [B3407] |
| **187** | v37427 | **1** | Norwegian krone, noon spot rate, average (dollars) [B3409] |
| **188** | v37428 | **1** | Swedish krona, noon spot rate, average (dollars) [B3410] |
| **189** | v37429 | **1** | Swiss franc, noon spot rate, average (dollars) [B3411] |
| **190** | v37430 | **1** | United Kingdom pound sterling, noon spot rate, average (dollars) [B3412] |
| **191** | v37431 | **1** | United Kingdom pound sterling, 90-day forward noon rate (dollars) [B3413] |
| **192** | v37432 | **1** | United States dollar, closing spot rate (dollars) [B3414] |
| **193** | v37433 | **1** | United States dollar, highest spot rate (dollars) [B3415] |
| **194** | v37434 | **1** | United States dollar, lowest spot rate (dollars) [B3416] |
| **195** | v37435 | **1** | United States dollar, 90-day forward closing rate (dollars) [B3417] |
| **196** | v41498903 | **1** | Canadian dollar effective exchange rate index (CERI) (1992=100) (dollars) |
| | | | **Table 176-0043 Interest rates** |
| **197** | v122550 | **1** | Bank rate, last Tuesday or last Thursday (percent) [B14079] |
| **198** | v122530 | **1** | Bank rate (percent) [B14006] |
| **199** | v122495 | **1** | Chartered bank administered interest rates - prime business (percent) [B14020] |
| **200** | v122505 | **1** | Forward premium or discount (-), United States dollar in Canada: 3 month (percent) [B14034] |
| **201** | v122509 | **1** | Prime corporate paper rate: 1 month (percent) [B14039] |
| **202** | v122556 | **1** | Prime corporate paper rate: 2 month (percent) [B14084] |
| **203** | v122491 | **1** | Prime corporate paper rate: 3 month (percent) [B14017] |
| **204** | v122504 | **1** | Bankers' acceptances: 1 month (percent) [B14033] |
| **205** | v122558 | **1** | Government of Canada marketable bonds, average yield: 1-3 year (percent) [B14009] |
| **206** | v122485 | **1** | Government of Canada marketable bonds, average yield: 3-5 year (percent) [B14010] |
| **207** | v122486 | **1** | Government of Canada marketable bonds, average yield: 5-10 year (percent) [B14011] |
| **208** | v122487 | **1** | Government of Canada marketable bonds, average yield: over 10 years (percent) [B14013] |
| **209** | v122515 | **1** | Chartered bank - 5 year personal fixed term (percent) [B14045] |
| **210** | v122493 | **1** | Chartered bank - non-chequable savings deposits (percent) [B14019] |
| **211** | v122541 | **1** | Treasury bill auction - average yields: 3 month (percent) [B14007] |
| **212** | v122484 | **1** | Treasury bill auction - average yields: 3 month, average at values (percent) [B14001] |
| **213** | v122552 | **1** | Treasury bill auction - average yields: 6 month (percent) [B14008] |
| **214** | v122554 | **1** | Treasury bills: 2 month (percent) [B14082] |
| **215** | v122531 | **1** | Treasury bills: 3 month (percent) [B14060] |
| **216** | v122499 | **1** | Government of Canada marketable bonds, average yield, average of Wednesdays: 1-3 year (percent) [B14028] |
| **217** | v122500 | **1** | Government of Canada marketable bonds, average yield, average of Wednesdays: 3-5 year (percent) [B14029] |
| **218** | v122502 | **1** | Government of Canada marketable bonds, average yield, average of Wednesdays: 5-10 year (percent) [B14030] |
| **219** | v122501 | **1** | Government of Canada marketable bonds, average yield, average of Wednesdays: over 10 years (percent) [B14003] |
| **220** | v122497 | **1** | Average residential mortgage lending rate: 5 year (percent) [B14024] |
| **221** | v122506 | **1** | Chartered bank - chequable personal savings deposit rate (percent) [B14035] |
| **222** | v122507 | **1** | Covered differential: Canada-United States 3 month Treasury bills (percent) [B14036] |
| **223** | v122508 | **1** | Covered differential: Canada-United States 3 month short-term paper (percent) [B14038] |
| **224** | v122510 | **1** | First coupon of Canada Savings Bonds (percent) [B14040] |
| | | | **Table 176-0051 Canada's official international reserves** |
| **225** | v122396 | **5** | Total, Canada's official international reserves (dollars - millions) [B3800] |
| **226** | v122397 | **5** | Convertible foreign currencies, United States dollars (dollars - millions) [B3801] |
| **227** | v122398 | **5** | Convertible foreign currencies, other than United States (dollars - millions) [B3802] |
| **228** | v122399 | **5** | Gold (dollars - millions) [B3803] |
| **229** | v122401 | **5** | Reserve position in the International Monetary Fund (IMF) (dollars - millions) [B3805] |

**Table 176-0032 Credit measures**

| | | | |
|---|---|---|---|
| **230** | v36414 | **5** | Total business and household credit; Seasonally adjusted (dollars - millions) [B165] |
| **231** | v36415 | **5** | Household credit; Seasonally adjusted (dollars - millions) [B166] |
| **232** | v36416 | **5** | Residential mortgage credit; Seasonally adjusted (dollars - millions) [B167] |
| **233** | v36417 | **5** | Consumer credit; Seasonally adjusted (dollars - millions) [B168] |
| **234** | v36418 | **5** | Business credit; Seasonally adjusted (dollars - millions) [B169] |
| **235** | v36419 | **5** | Other business credit; Seasonally adjusted (dollars - millions) [B170] |
| **236** | v36420 | **5** | Short-term business credit; Seasonally adjusted (dollars - millions) [B171] |

**Table 176-0025 Monetary aggregates**

| | | | |
|---|---|---|---|
| **237** | v37148 | **5** | Currency outside banks (dollars - millions) [B1604] |
| **238** | v37153 | **5** | Canadian dollar assets, total loans (dollars - millions) [B1605] |
| **239** | v37154 | **5** | General loans (including grain dealers and installment finance companies) (dollars - millions) [B1606] |
| **240** | v37107 | **5** | Total, major assets (dollars - millions) [B1611] |
| **241** | v37111 | **5** | Canadian dollar assets, liquid assets (dollars - millions) [B1615] |
| **242** | v37112 | **5** | Canadian dollar assets, less liquid assets (dollars - millions) [B1616] |
| **243** | v37119 | **5** | Total personal loans, average of Wednesdays (dollars - millions) [B1622] |
| **244** | v37120 | **5** | Business loans, average of Wednesdays (dollars - millions) [B1623] |
| **245** | v41552793 | **5** | Currency outside banks and chartered bank deposits, held by general public (including private sector float) (dollars - millions) |
| **246** | v41552795 | **5** | M1B (gross) (currency outside banks, chartered bank chequable deposits, less inter-bank chequable deposits) (dollars - millions) |
| **247** | v41552796 | **5** | M2 (gross) (currency outside banks, chartered bank demand and notice deposits, chartered bank personal term deposits, adjustments to M2 (gross) (continuity adjustments and inter-bank demand and notice deposits)) (dollars - millions) |
| **248** | v41552797 | **5** | Currency outside banks and chartered bank deposits (including private sector float) (dollars - millions) |
| **249** | v37130 | **5** | Residential mortgages (dollars - millions) [B1632] |
| **250** | v41552798 | **5** | M2+ (gross) (dollars - millions) |
| **251** | v37135 | **5** | Chartered bank deposits, personal, term (dollars - millions) [B1637] |
| **252** | v37138 | **5** | Total, deposits at trust and mortgage loan companies (dollars - millions) [B1639] |
| **253** | v37139 | **5** | Total, deposits at credit unions and caisses populaires (dollars - millions) [B1640] |
| **254** | v37140 | **5** | Bankers' acceptances (dollars - millions) [B1641] |
| **255** | v37145 | **5** | Monetary base (notes and coin in circulation, chartered bank and other Canadian Payments Association members' deposits with the Bank of Canada) (dollars - millions) [B1646] |
| **256** | v37146 | **5** | Monetary base (notes and coin in circulation, chartered bank and other Canadian Payments Association members' deposits with the Bank of Canada) (excluding required reserves) (dollars - millions) [B1647] |
| **257** | v37147 | **5** | Canada Savings Bonds and other retail instruments (dollars - millions) [B1648] |
| **258** | v41552801 | **5** | M2++ (gross) (M2+ (gross), Canada Savings Bonds, non-money market mutual funds) (dollars - millions) |
| **259** | v37152 | **5** | M1++ (gross) (dollars - millions) [B1652] |

**Table 282-0087 LFS, SA, Canada and provinces**

| | | | |
|---|---|---|---|
| **260** | v2062811 | **5** | Canada; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands) |
| **261** | v2062815 | **1** | Canada; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate) |
| **262** | v2063000 | **5** | Newfoundland and Labrador; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands) |
| **263** | v2063004 | **1** | Newfoundland and Labrador; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate) |
| **264** | v2063189 | **5** | Prince Edward Island; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands) |
| **265** | v2063193 | **1** | Prince Edward Island; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate) |
| **266** | v2063378 | **5** | Nova Scotia; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands) |
| **267** | v2063382 | **1** | Nova Scotia; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate) |
| **268** | v2063567 | **5** | New Brunswick; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands) |
| **269** | v2063571 | **1** | New Brunswick; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate) |
| **270** | v2063756 | **5** | Quebec; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands) |
| **271** | v2063760 | **1** | Quebec; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate) |
| **272** | v2063945 | **5** | Ontario; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands) |
| **273** | v2063949 | **1** | Ontario; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate) |
| **274** | v2064134 | **5** | Manitoba; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands) |
| **275** | v2064138 | **1** | Manitoba; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate) |
| **276** | v2064323 | **5** | Saskatchewan; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands) |
| **277** | v2064327 | **1** | Saskatchewan; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate) |
| **278** | v2064512 | **5** | Alberta; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands) |
| **279** | v2064516 | **1** | Alberta; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate) |
| **280** | v2064701 | **5** | British Columbia; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands) |
| **281** | v2064705 | **1** | British Columbia; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate) |

**Table 282-0088 Employment by industry**

| | | | |
|---|---|---|---|
| **282** | v2057603 | **5** | Total employed, all industries; Seasonally adjusted (persons - thousands) |
| **283** | v2057604 | **5** | Goods-producing sector; Seasonally adjusted (persons - thousands) |
| **284** | v2057605 | **5** | Agriculture [1100-1129, 1151-1152]; Seasonally adjusted (persons - thousands) |
| **285** | v2057606 | **5** | Forestry, fishing, mining, oil and gas [1131-1133, 1141-1142, 1153, 2100-2131]; Seasonally adjusted (persons - thousands) |
| **286** | v2057607 | **5** | Utilities [2211-2213]; Seasonally adjusted (persons - thousands) |
| **287** | v2057608 | **5** | Construction [2361-2389]; Seasonally adjusted (persons - thousands) |
| **288** | v2057609 | **5** | Manufacturing [3211-3219, 3271-3279, 3311-3399, 3111-3169, 3221-3262]; Seasonally adjusted (persons - thousands) |
| **289** | v2057610 | **5** | Services-producing sector; Seasonally adjusted (persons - thousands) |
| **290** | v2057611 | **5** | Trade [4111-4191, 4411-4543]; Seasonally adjusted (persons - thousands) |
| **291** | v2057612 | **5** | Transportation and warehousing [4811-4931]; Seasonally adjusted (persons - thousands) |
| **292** | v2057613 | **5** | Finance, insurance, real estate and leasing [5211-5269, 5311-5331]; Seasonally adjusted (persons - thousands) |
| **293** | v2057614 | **5** | Professional, scientific and technical services [5411-5419]; Seasonally adjusted (persons - thousands) |
| **294** | v2057615 | **5** | Business, building and other support services [5511-5629]; Seasonally adjusted (persons - thousands) |
| **295** | v2057616 | **5** | Educational services [6111-6117]; Seasonally adjusted (persons - thousands) |
| **296** | v2057617 | **5** | Health care and social assistance [6211-6244]; Seasonally adjusted (persons - thousands) |
| **297** | v2057618 | **5** | Information, culture and recreation [5111-5191, 7111-7139]; Seasonally adjusted (persons - thousands) |
| **298** | v2057619 | **5** | Accommodation and food services [7211-7224]; Seasonally adjusted (persons - thousands) |
| **299** | v2057620 | **5** | Other services [8111-8141]; Seasonally adjusted (persons - thousands) |
| **300** | v2057621 | **5** | Public administration [9110-9191]; Seasonally adjusted (persons - thousands) |

| | | | Tables 228-00(01,41) Merchandise imports and exports Canada, SA |
|---|---|---|---|
| **301** | v183474 | **5** | Imports, United States, including Puerto Rico and Virgin Islands (dollars - millions) [D398058] |
| **302** | v183475 | **5** | Imports, United Kingdom (dollars - millions) [D398059] |
| **303** | v183476 | **5** | Imports, Other European Economic Community (dollars - millions) [D398060] |
| **304** | v183477 | **5** | Imports, Japan (dollars - millions) [D398061] |
| **305** | v191559 | **5** | Exports, United States, including Puerto Rico and Virgin Islands (dollars - millions) [D399518] |
| **306** | v191560 | **5** | Exports, United Kingdom (dollars - millions) [D399519] |
| **307** | v191561 | **5** | Exports, Other European Economic Community (dollars - millions) [D399520] |
| **308** | v191562 | **5** | Exports, Japan (dollars - millions) [D399521] |
| **309** | v21386488 | **5** | Imports, total of all merchandise (dollars - millions) |
| **310** | v21386489 | **5** | Imports, Sector 1 Agricultural and fishing products (dollars - millions) |
| **311** | v21386492 | **5** | Imports, Sector 2 Energy products (dollars - millions) |
| **312** | v21386495 | **5** | Imports, Sector 3 Forestry products (dollars - millions) |
| **313** | v21386496 | **5** | Imports, Sector 4 Industrial goods and materials (dollars - millions) |
| **314** | v21386500 | **5** | Imports, Sector 5 Machinery and equipment (dollars - millions) |
| **315** | v21386505 | **5** | Imports, Sector 6 Automotive products (dollars - millions) |
| **316** | v21386509 | **5** | Imports, Sector 7 Other consumer goods (dollars - millions) |
| **317** | v21386512 | **5** | Imports, Sector 8 Special transactions trade (dollars - millions) |
| **318** | v21386514 | **5** | Exports, total of all merchandise (dollars - millions) |
| **319** | v21386515 | **5** | Exports, Sector 1 Agricultural and fishing products (dollars - millions) |
| **320** | v21386518 | **5** | Exports, Sector 2 Energy products (dollars - millions) |
| **321** | v21386522 | **5** | Exports, Sector 3 Forestry products (dollars - millions) |
| **322** | v21386526 | **5** | Exports, Sector 4 Industrial goods and materials (dollars - millions) |
| **323** | v21386531 | **5** | Exports, Sector 5 Machinery and equipment (dollars - millions) |
| **324** | v21386535 | **5** | Exports, Sector 6 Automotive products (dollars - millions) |
| **325** | v21386539 | **5** | Exports, Sector 7 Other consumer goods (dollars - millions) |
| **326** | v21386540 | **5** | Exports, Sector 8 Special transactions trade (dollars - millions) |
| | | | **Table 026-0008: Building permits, values by activity sector; Canada** |
| **327** | v4667 | **5** | Total residential and non-residential (dollars - thousands) [D2677] |
| **328** | v4668 | **5** | Residential (dollars - thousands) [D2681] |
| **329** | v4669 | **5** | Non-residential (dollars - thousands) [D4898] |
| **330** | v4670 | **5** | Industrial (dollars - thousands) [D2678] |
| **331** | v4671 | **5** | Commercial (dollars - thousands) [D2679] |
| **332** | v4672 | **5** | Institutional and governmental (dollars - thousands) [D2680] |

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