Logic and hypothesis testing : reflexions on ill-defined problems problems in econometrics*

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1. Introduction

Recent developments in econometrics :

1. new fields of applications linked to the availability of new data, financial data, micro-data, panels, qualitative variables, etc. ;
2. great variety of new models : multivariate time series models, GARCH, etc. ;
3. greater ability to estimate nonlinear models which require an important computational capacity ;
4. methods based on simulation : bootstrap, indirect inference, etc. ;
5. methods based on weak distributional assumptions : nonparametric methods, asymptotic distributions based on "weak regularity conditions", etc. ;
6. discovery of various nonregular problems which require nonstandard distributional theories : unit roots, etc.

Two types of ill-defined problems in econometrics :

1. trying to solve an inference problem using a technique that cannot deliver a solution **because of the very structure of the technique** :
   two examples :
   (a) in the context of a structural model, to build a confidence interval for a parameter which is not identifiable, using the usual technique based on standard errors ;
   (b) to test an hypothesis on a mean under an assumption heteroskedasticity of unknown form, using the usual techniques based on correcting least square standard errors ("heteroskedasticity-robust methods") ;
2. trying to solve a statistical problem for which no reasonable no reasonable solution can possibly exist :

(a) testing an hypothesis on a dynamic model, allowing a dynamic structure (under the null hypothesis) which involves an unlimited (not necessarily infinite) number of parameters ;

(b) testing an hypothesis on a mean in the context of a nonparametric model, *e.g.* assuming that the observations are i.i.d. with a finite mean.

2. Models and hypotheses

The purpose of econometric analysis is to develop mathematical representations of data, which we call **models** or **hypotheses** (models subject to restrictions).
An hypothesis must have two basic properties :

1. to restrict the expected behavior of observations : to be **informative ;**
   an non-restrictive hypothesis says nothings and, consequently, does not learn us anything : it is

   empirically empty ,
   void **of empirical content ;**

   the more restrictive a model is, the more informative it is, the more interesting it is ;
2. to be **compatible with available data** ;
   ideally, we would like it to be **true**.

These two criteria oppose each other :

1. **information criterion** $\longrightarrow$ **parsimony** $\longrightarrow$ parametric models, strong assumptions ;
2. **compatibility with observed data** $\longrightarrow$ **vague models**, little restrictive $\longrightarrow$ nonparametric models, weak hypotheses.

There is a wide current of thought in philosophy of science that emphasizes

falsifiability as a criterion for the scientific character of a theory (Popper).

Deterministic models (claim to make arbitrarily precise predictions) :

– highly falsifiable ;
– always inconsistent with observed data.

Probabilistic models

Most models used in econometrics are **probabilistic**, which has two consequences :

1. **unverifiable** :
   as for any theory that makes an indefinite number of predictions, we can never be sure that the model will not be jeopardized by new data ;
2. **logically unfalsifiable** : (in contrast with deterministic models) :
   a probabilistic model is usually logically compatible with all possible observation vectors.

Given these facts, it is clear any criterion for assessing whether an hypothesis is acceptable must involve a conventional aspect.

The purpose of hypothesis testing theory is to supply a coherent framework for accepting or rejecting probabilistic hypotheses.

It is a probabilistic adaptation of the falsification principle.

3. Statistical inference

Consider an experiment whose result can be represented by a vector of observations

$$\mathbf{X}^{(n)} = (X_1\,,\,\ldots\,,\,X_n)' \tag{3.1}$$

where X_i takes real values, and let

$$\overline{F}_n(x_1\,,\,\ldots\,,\,x_n) = P[X_1 \leq x_1\,,\,\ldots\,,\,X_n \leq x_n] \tag{3.2}$$

its distribution.

Let

$$\mathcal{F}_n = \{\text{Functions de distribution on } \mathbb{R}^n\}\;. \tag{3.3}$$

An hypothesis H_0 for $X^{(n)}$ is an assertion which states that

$$H_0 : \overline{F}_n \in \mathcal{H}_0 \tag{3.4}$$

where H_0 is a subset of all possible distributions on F_n.
In particular, H_0 often takes the following form :

$$\mathcal{H}_0 = \mathcal{H}(F_0\,,\theta_1^0) \equiv \{F(x)\;,\; x \in \mathbb{R}^n : F(x) = F_0(x\,|\,\theta_1, \theta_2) \text{ and } \theta_1 = \theta_1^0\} \tag{3.5}$$

where F_0 is a function with a specific form [e.g., corresponding to a Gaussian law] and $(\theta_1,\, \theta_2) \in \Omega_1 \times \Omega_2$. In this case, we can write in short form : :

$$H_0 : \theta_1 = \theta_1^0\;. \tag{3.6}$$

where :

θ_1 is the parameter of interest,

θ_2 is a nuisance parameter.

H_0 may contain single distribution (*simple hypothesis*) or several distributions (*composite hypothesis*).
Strong interpretation of H_0 : the "true" distribution of $X^{(n)}$ belongs to H_0 .

Weak interpretation of H_0 : there is at least one distribution in H_0 that can be viewed as a representation compatible the observed "behavior" of $X^{(n)}$.
Irrespective of the interpretation, we have :

$$H_0 \text{ is acceptable } \Longleftrightarrow \Big((\exists F \in \mathcal{H}_0)\ F \text{ is acceptable}\Big) \tag{3.7}$$

$$\Big(\sim \big((\exists F \in \mathcal{H}_0)\ F \text{ is acceptable}\big)\Big) \Longleftrightarrow \Big((\forall F \in \mathcal{H}_0)\quad F \text{ is inacceptable}\Big) \tag{3.8}$$

$$H_0 \text{ is inacceptable } \Longleftrightarrow \Big((\forall F \in \mathcal{H}_0)\quad F \text{ is inacceptable}\Big) \tag{3.9}$$

A test for H_0 is a rule by which one decides to reject or accept the hypothesis (or to view it as incompatible with the data).
In general, we can represent the rule using an indicator function $\phi(X_1, \ldots, X_n)$ which takes the values 0 or 1 :

$$\begin{aligned} \phi(X_1, \ldots, X_n) &= 1 \quad \text{means that } H_0 \text{ is rejected,} \\ &= 0 \quad \text{means that } H_0 \text{ is accepted.} \end{aligned} \tag{3.10}$$

By definition, $\phi(X_1, \ldots, X_n)$ has level α iff

$$\mathsf{E}_F[\phi(X_1, \ldots, X_n)] = \mathsf{P}_F[\text{Rejecting } H_0] \leq \alpha \text{ for all } F \in \mathcal{H}_0 \tag{3.11}$$

or, equivalently,

$$\sup_{F \in \mathcal{H}_0} \mathsf{P}_F[\text{Rejecting } H_0] \leq \alpha \tag{3.12}$$

Usually, $\phi(X_1, \ldots, X_n)$ is defined in the following way :

$$\begin{aligned} \phi(X_1, \ldots, X_n) &= 1 \quad \text{if } S_n(X_1, \ldots, X_n) > c \\ &= 0 \quad \text{if } S_n(X_1, \ldots, X_n) \leq c \end{aligned} \tag{3.13}$$

If we consider an hypothesis of the form

$$H_0(\theta_1^0) : \theta_1 = \theta_1^0\ . \tag{3.14}$$

and if we can build a different test for each possible value of θ_1^0,

$$\begin{aligned} \phi(\theta_1^0; X_1, \ldots, X_n) &= 1 \quad \text{if } S_n(\theta_1^0; X_1, \ldots, X_n) > c(\theta_1^0) \\ &= 0 \quad \text{if } S_n(\theta_1^0; X_1, \ldots, X_n) \le c(\theta_1^0) \end{aligned} \tag{3.15}$$

we can determine the set of values which are can be viewed as compatible with the data according to the tests considered :

$$C = \left\{\theta_1^0 : \phi(\theta_1^0 ; X_1, \ldots, X_n) = 0\right\} . \tag{3.16}$$

If

$$\mathsf{E}_F[\phi(\theta_1^0 ; X_1, \ldots, X_n)] \le \alpha \quad \text{for all} \quad F \in \mathcal{H}(F_0, \theta_1^0) \tag{3.17}$$

we have

$$\inf_{\theta_1, \theta_2} \mathsf{P}[\theta_1 \in C] \ge 1 - \alpha . \tag{3.18}$$

C is a confidence region with level $1 - \alpha$ for θ_1.

In practice, *confidence regions* (or *confidence intervals*) were made possible by the discovery of pivotal statistics :

$$S_n(\theta_1; X_1, \ldots, X_n) \sim \text{Distribution without nuisance parameters (or boundable).} \tag{3.19}$$

We can find a point c such that :

$$\mathsf{P}[S_n(\theta_1; X_1, \ldots, X_n) > c] \ge \alpha , \ \forall \theta_1 . \tag{3.20}$$

Difficulties : there are problems where :

1. the proposed statistics may not may not be pivotal ;
2. there is no valid test that satisfies reasonable properties [e.g., to depend upon the data] :

non testable hypothesis ,

empirically **empty hypothesis** .

4. Inference on structural models and weak instruments

Several authors in the pas have noted that usual asymptotic approximations are not valid or lead to very inaccurate results when parameters of interest are close to regions where these parameters are not anymore identifiable :

Sargan (1983, Econometrica)
Phillips (1984, International Economic review)
Phillips (1985)
Gleser and Hwang (1987, Annals of Statistics)
Koschat (1987, Annals of Statistics)
Phillips (1989, Econometric Theory)
Hillier (1990, Econometrica)
Nelson and Startz (1990a, Journal of Business)
Nelson and Startz (1990b, Econometrica)
Buse (1992, Econometrica)
Maddala and Jeong (1992, Econometrica)
Choi and Phillips (1992, Journal of Econometrics)
Bound, Jaeger, and Baker (1993, NBER Discussion Paper)
Dufour and Jasiak (1993, CRDE)
Bound, Jaeger, and Baker (1995, Journal of the American Statistical Association)
McManus, Nankervis, and Savin (1994, Journal of Econometrics)
Hall, Rudebusch, and Wilcox (1996, International Economic Review)
Dufour (1997, Econometrica)
Shea (1997, Review of Economics and Statistics)
Staiger and Stock (1997, Econometrica)
Wang and Zivot (1998, Econometrica)
Zivot, Startz, and Nelson (1998, International Economic Review)
Dufour and Jasiak (1999, International Economic Review, à paraître)
Startz, Nelson, and Zivot (1999, International Economic Review)

Consider a situation where we have two parameters θ_1 and θ_2 such that θ_2 stops being identifiable when θ_1 takes a certain value, say $\theta_1 = \theta_1^0$:

$$L(y|\theta_1, \theta_2) = \overline{L}(y|\theta_1^0) \quad \text{does not depend on } \theta_2 \text{ when} \quad \theta_1 = \theta_1^0 \,. \tag{4.1}$$

4.1 Theorem *If θ_2 is a parameter whose value is not bounded, then the confidence region C with level $1-\alpha$ for θ_2 must have the following property :*

$$\mathsf{P}[C \text{ is unbounded}] > 0 \tag{4.2}$$

and, if $\theta_1 = \theta_1^0$,

$$\mathsf{P}[C \text{ is unbounded}] \geq 1 - \alpha \,. \tag{4.3}$$

DÉMONSTRATION. See Dufour (1997, Econometrica). ∎

4.2 Corollary *If C does not satisfy the property given in the previous theorem, its level must be zero.*

This will be the case, in particular, for any Wald-type confidence interval, obtained by assuming that

$$t_{\widehat{\theta}_2} = \frac{\widehat{\theta}_2 - \theta_2}{\widehat{\sigma}_{\theta_2}} \overset{approx}{\sim} N(0,1) \quad \text{[or another distribution]} \tag{4.4}$$

hence an interval of the form

$$\widehat{\theta}_2 - c\widehat{\sigma}_{\theta_2} \leq \theta_2 \leq \widehat{\theta}_2 + c\widehat{\sigma}_{\theta_2} \tag{4.5}$$

where $P[|N(0,1)| > c] \leq \alpha$. This interval has level :

$$\inf_{\theta} \mathsf{P}\left[\widehat{\theta}_2 - c\widehat{\sigma}_{\theta_2} \leq \theta_2 \leq \widehat{\theta}_2 + c\widehat{\sigma}_{\theta_2}\right] = 0 \,. \tag{4.6}$$

In many models, the notion of standard error loses its usual meaning and does not constitute a valid basis for building confidence intervals.

5. Inference on nonparametric models

5.1. Procedures robust to heteroskedasticity of unknown form

$$H_0 : \quad X_1, \ldots, X_n \text{ are independent observations each one with a distribution symmetric about zero.} \tag{5.1}$$

H_0 allows arbitrary heteroskedasticity. Let

$$\mathcal{H}_0 = \{F \in \mathcal{F}_n : F \text{ satisfies } H_0\} \tag{5.2}$$

5.1 Theorem *If $\phi(X_1, \ldots, X_n)$ is a test with level α for H_0, where $0 \leq \alpha < 1$, then $\phi(X_1, \ldots, X_n)$ must satisfy the condition*

$$\mathsf{E}[\phi(X_1, \ldots, X_n) \mid |X_1|, \ldots, |X_n|] \leq \alpha \text{ under } H_0 . \tag{5.3}$$

DÉMONSTRATION. See Pratt and Gibbons (1981, Concepts of Nonparametric Theory, Section 5.10) and Lehmann and Stein (1949, Annals of Mathematical Statistics). ∎

$\phi(X_1, \ldots, X_n)$ must be a **sign test** (or, more generally, a sign test conditional on the absolute values of the observations).

5.2 Corollary *If, for all $0 \leq \alpha < 1$, the condition (5.3) is not satisfied, then the size of the test $\phi(X_1, \ldots, X_n)$ is equal to one, i.e.*

$$\sup_{F_n \in \mathcal{H}_0} \mathsf{E}_{F_n}[\phi(X_1, \ldots, X_n)] = 1 . \tag{5.4}$$

All test procedures typically designated as "robust to heteroskedasticity" (White-type) do not satisfy condition (5.3) and consequently have size one :

For examples of size distortions, see :

Dufour (1981, Journal of Time Series Analysis),

Campbell and Dufour (1995, Review of Economics and Statistics), Campbell and Dufour (1997, International Economic Review).

5.2. Procedures robust to autocorrelation of arbitrary form

Consider the problem of testing the unit root hypothesis in the context of an autoregressive model whose order is infinite or in not bounded by a prespecified maximal order :

$$X_t \;=\; \beta_0 + \sum_{k=1}^{p} \lambda_k X_{t-k} + u_t \;,\; t = 1\,,\;\ldots\;,\; T \tag{5.5}$$

$$u_t \overset{i.i.d.}{\sim} N[0\,,\; \sigma^2] \tag{5.6}$$

where p is not bounded *a priori*. We wish to test :

$$H_0 : \sum_{k=1}^{p} \lambda_k = 1 \;. \tag{5.7}$$

$$\begin{gathered} H_0 :\;\; X_t = \beta_0 + \sum_{k=1}^{p} \lambda_k X_{t-k} + u_t \;,\; .t = 1\,,\;\ldots\;,\; T\,, \\ \sum_{k=1}^{p} \lambda_k = 1 \text{ and } u_t \overset{i.i.d.}{\sim} N[0\,,\; \sigma^2] \end{gathered} \tag{5.8}$$

5.3 Theorem *If $\phi(X_1, \ldots, X_n)$ is a test with level α for H_0, i.e.*

$$\mathsf{P}_F[\text{Rejecting } H_0] = E_F[\phi(X_0\,, X_1, \ldots, X_n)] \le \alpha \quad \text{for all } F \text{ satisfying } H_0\;, \tag{5.9}$$

then,

$$\mathsf{P}_F[\text{Rejecting } H_0] = E_F[\phi(X_1, \ldots, X_n)] \le \alpha \text{ for all } F \in \mathcal{H}_0\;. \tag{5.10}$$

Démonstration. See Cochrane (1991, Journal of Economic Dynamics and Control) and Blough (1992, Journal of Applied Econometrics). ∎

The test must behave in the following way (for a test of level .05) :

1. we throw away all data to garbage ;

2. using a random number generator, produce a realization of $U \sim U(0,\ 1)$;
3. reject H_0 if $U \leq .05$.

Testable hypothesis :

$$\begin{aligned} H_0(6):\ \ X_t = \beta_0 + \sum_{k=1}^{6}\lambda_k X_{t-k} + u_t\ ,\ .t = 1\,,\ \ldots\ ,\ T\,, \\ \sum_{k=1}^{6}\lambda_k = 1 \text{ and } u_t \overset{i.i.d.}{\sim} N[0\,,\ \sigma^2] \end{aligned} \tag{5.11}$$

Similar difficulties for most hypotheses on the coefficients of (5.8).

Other relevant references : Sims (1971a), Sims (1971b), Blough (1992), Faust (1996), Faust (1999).

5.3. Procedures robust to nonnormality

Bahadur and Savage (1956)
Tibshirani and Wasserman (1988, Canadian Journal of Statistics)

$$H_0(\mu): \; X_1\,,\,\ldots\,,\,X_n \text{ are i.i.d. observations such that } E(X_1) = \mu \tag{5.12}$$

We wish to test the hypothesis that $X_1\,,\,\ldots\,,\,X_n$ have mean zero, under the general assumption that the observations $X_1\,,\,\ldots\,,\,X_n$ are i.i.d. Let

$$\mathcal{H}(\mu) = \{\text{Distribution functions} \quad F \in \mathcal{F}_n \text{ such that } H_0(\mu) \text{ is satisfied}\}\,. \tag{5.13}$$

5.4 Theorem *If $\phi(X_1,\,\ldots\,,\,X_n)$ is a test with level α for $H_0(\mu_0)$, i.e.*

$$P_F[\textit{Rejecting } H_0(\mu_0)] = E_F[\phi(X_1,\,\ldots\,,\,X_n)] \leq \alpha \textit{ for all } F \in \mathcal{H}(\mu_0)\,, \tag{5.14}$$

then, for any $\mu \neq \mu_0$,

$$P_F[\textit{Rejecting } H_0(\mu_0)] = E_F[\phi(X_1,\,\ldots\,,\,X_n)] \leq \alpha \textit{ for all } F \in \mathcal{H}(\mu)\,. \tag{5.15}$$

DÉMONSTRATION. See Bahadur and Savage (1956). ∎

5.5 Theorem *If $\phi(X_1,\,\ldots\,,\,X_n)$ is a test with level α for $H_0(\mu_0)$, i.e.*

$$P_F[\textit{Rejecting } H_0(\mu_0)] = E_F[\phi(X_1,\,\ldots\,,\,X_n)] \leq \alpha \textit{ for all } F \in \mathcal{H}(\mu_0) \tag{5.16}$$

and, if there one value $\mu_1 \neq \mu_0$ such that

$$P_F[\textit{Rejecting } H_0(\mu_0)] = E_F[\phi(X_1,\,\ldots\,,\,X_n)] \geq \alpha \textit{ pout } F \in \mathcal{H}(\mu_1)\,, \tag{5.17}$$

then, for any $\mu \neq \mu_0$,

$$P_F[\textit{Rejecting } H_0(\mu_0)] = E_F[\phi(X_1, \ldots, X_n)] = \alpha \textit{ for all } F \in \mathcal{H}(\mu) . \tag{5.18}$$

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