# Confidence regions for calibrated parameters in computable general equilibrium models [*]

Touhami Abdelkhalek [†]
I.N.S.E.A., Rabat

Jean-Marie Dufour [‡]
McGill University

First version: November 1997
Revised: August 1998, November 2001
This version: October 2003
Compiled: September 18, 2007 12:47pm

This article has been published in the *Annales d'Économie et de Statistique*, 81 (2006), 1-32.

[*] The authors thank Bryan Campbell, Bernard Decaluwé, René Garcia, André Martens, Nour Meddahi, Randall Wigle, two anonymous referees, and the Editor Franck Portier for several useful comments. We are also very grateful to Paul Klassen for his translation and edition work. This research was conducted under the auspices of the PARADI program linking two research centers, C.R.D.E. (Université de Montréal) and CREFA (Université Laval), and financed as a Center of Excellence in International Development by the Canadian Agency for International Development. This work was also supported the Canada Research Chair Program (Chair in Econometrics, Université de Montréal), the Institut de Finance mathématique de Montréal (IFM2), the Canadian Network of Centres of Excellence [program on *Mathematics of Information Technology and Complex Systems*, (MITACS)], the Fonds de recherche sur la société et la culture (Québec), the Fonds de recherche sur la nature et les technologies (Québec), the Natural Sciences and Engineering Research Council of Canada, the Social Sciences and Humanities Research Council of Canada, the Canada Council for the Arts (Killam Fellowship), and the Alexander-von-Humboldt Foundation (Germany).

[†] I.N.S.E.A., B.P. 6217, Rabat-Instituts, Rabat, Morocco. TEL: 212 37 77 09 26; FAX: 212 37 77 94 57; e-mail: Atouhami@insea.ac.ma .

[‡] William Dow Professor of Economics, McGill University, Centre interuniversitaire de recherche en analyse des organisations (CIRANO), and Centre interuniversitaire de recherche en économie quantitative (CIREQ). Mailing address: Department of Economics, McGill University, Leacock Building, Room 519, 855 Sherbrooke Street West, Montréal, Québec H3A 2T7, Canada. TEL: (1) 514 398 8879; FAX: (1) 514 398 4938; e-mail: jean-marie.dufour@mcgill.ca . Web page: http://www.jeanmariedufour.com

# Abstract

We consider the problem of assessing the uncertainty of calibrated parameters in computable general equilibrium (CGE) models through the construction of confidence sets (or intervals) for these parameters. We study two different setups under which this can be done. The first one extends earlier work from Abdelkhalek and Dufour (1998) and is based on a projection technique which allows the construction of confidence sets for calibrated parameters from confidence sets on the free parameters of a (deterministic) CGE model. We discuss in detail how this approach can be applied to CES (Armington-type) function parameters frequently used in CGE models and illustrate it on models of the Moroccan economy. The second method allows one to extend the usual deterministic specification of CGE models by adding stochastic disturbances to equations of the model and then to construct corresponding confidence sets for calibrated parameters using simulation techniques. This method uses the classical concept of a pivotal function for a parameter. We discuss in detail how this method can be applied to the calibrated parameters of a Cobb-Douglas production function. Applications to CGE models of the Moroccan economy are presented.

**Keywords**: computable general equilibrium model; calibration; sensitivity analysis; confidence set; confidence interval; projection; Morocco.

**Journal of Economic Literature classification:** C100, C190, C500, C510, C520, O210.

# Résumé

Nous considérons le problème de la prise en compte de l'incertitude sur les paramètres calibrés de modèles calculables d'équilibre général (MCEG) en construisant des régions (ou des intervalles) de confiance pour ces paramètres. Nous étudions en détail deux méthodes qui permettent de ce faire. La première est une extension des travaux de Abdelkhalek et Dufour (1998) et repose sur une technique de projection qui permet de construire des régions de confiance pour les paramètres calibrés à partir de régions de confiance pour les paramètres libres d'un MCEG déterministe. Nous discutons en détail comment cette approche peut être appliquée aux paramètres d'une fonction CES (de type Armington) d'usage fréquent dans les MCEG et nous l'illustrons sur des modèles de l'économie marocaine. La seconde méthode permet de dépasser le cadre déterministe usuel des MCEG en ajoutant des perturbations aléatoires à certaines équations du modèle pour construire des régions de confiance pour les paramètres calibrés en utilisant des techniques de simulation. Cette méthode utilise aussi le concept classique de fonction pivotale d'un paramètre. Nous discutons en détail comment cette méthode peut être appliquée aux paramètres calibrés d'une fonction de production de type Cobb-Douglas. Des applications à des MCEG de l'économie marocaine sont présentées.

**Mots clés**: modèle calculables d'équilibre général; calibration; région de confiance; intervalle de confiance; projection; analyse de sensibilité; Maroc.

**Classification du Journal of Economic Literature:** C100, C190, C500, C510, C520, O210.

# Contents



# List of Tables

# 1. Introduction

Computable general equilibrium (CGE) models are extensively used for analyzing and simulating the effects of economic policy changes in developing and industrialized countries. Presentations and overviews of this policy analysis tool may be found in Shoven and Whalley (1984, 1992), Manne (1985), Devarajan, Lewis and Robinson (1986, 1994), Martens (1993), Decaluwé and Martens (1988) as well as Gunning and Keyzer (1995). These models are generally non-stochastic and strongly nonlinear. Results obtained by simulating these models rely on several assumptions, pertaining both to the behavior of agents and to the choice of exogenous variables (the "closure" of the model). The nature and quality of the available data also affect the results, whether these are base-year data in static models (*e.g.*, the reference year in the social-accounting matrix) or the stationary equilibrium in dynamic models. The values assigned to the parameters of the behavioral functions, which underlie the "calibration" of the model, are also crucial. In fact, since the work of Mansur and Whalley (1984) and even before, CGE model designers have widely relied on calibration methods; for a review, see Dawkins, Srinivasan and Whalley (2001). These generally require a good deal less time and effort than econometric estimation. Calibration relies on a largely arbitrary distinction between "free parameters", which can be obtained from external sources or simply assigned on the basis of subjective judgements, and "*calibrated parameters*" which are derived ("estimated") from the former so as to reproduce the reference data (*e.g.,* base-year data). In these methods there clearly is a level of *uncertainty* associated with the selection of free parameters, since the calibration process depends on them.

The issue of the choice of values for the parameters of CGE models often gives rise to a natural scepticism among those who need to build, analyze, or use such models. In general, these values may be econometric estimates drawn from other studies, figures based on international comparisons, or simply arbitrary values imputed with no supporting data. Elasticities available in the literature are often contradictory and inconsistent. Frequently, they are obtained using sectorial classifications different from those of the model, and pertain to other time periods or countries. The varying degrees of uncertainty affecting these models transfers to the results of the simulations [see Abdelkhalek and Dufour (1998)]. Since CGE models are rarely estimated using econometric methods [except in the notable work of Jorgenson (1984) and his associates], it is *difficult to perform tests* on the data or build *confidence regions* for the calibrated parameters and the endogenous variables of the model. Even if the general specification of the model is not questioned, the credibility of the conclusions suffers from the uncertainty associated with reference-year data and parameter values.

On the latter source of uncertainty, Mansur and Whalley (1984, pp. 100 and 103) – among others – emphasize the crucial nature of the latter step in the model building process: "The choice of elasticity values critically affects results obtained with these models" and "The set of elasticity values used are critical parameters in determining the general equilibrium impacts of policy changes generated by these models". Shoven and Whalley (1984) in an article summarizing the main studies realized up to 1984 recognize the key role played by the selection of these parameters in determining economic policy simulations as well as the difficulties encountered by researchers during calibration. They indicate that the method generally used is based upon an arbitrary choice of point estimate around which sensitivity analysis may be performed. In particular, they write: "The procedure generally employed is to choose a central case specification, around which sensitivity analysis can be performed" [Shoven and Whalley (1984, pp. 1030–1031)]. On these issues, see also the review of Dawkins et al. (2001).

Recognizing the seriousness of this problem, a number of authors have proposed an assortment of approaches in order to translate parameter uncertainty into a measure of uncertainty for the simulated endogenous variables of the model (although *not for calibrated parameters*); see Pagan and Shannon (1985, 1989), Harrison (1986, 1989), Bernheim, Scholz and Shoven (1989), Wigle (1986, 1991), Harrison and Vinod (1992), Harrison, Jones, Kimbell and Wigle (1993). These methods are fundamentally descriptive and do not resort to a rigorous statistical framework. More recently, however, we proposed a more systematic approach, which allows the construction of confidence regions for the *endogenous variables* of CGE models in order to account for free-parameter uncertainty; see Abdelkhalek and Dufour (1998).

Calibration may be viewed as a two-stage estimation procedure by which, starting from the values of the free parameters and reference-year data, values are assigned to the calibrated parameters. This method, which is widely used in studies based on CGE models, has the advantage of being much less demanding than traditional econometric methods, both from the perspective of data requirements and numerical procedures.

Like free parameters, calibrated parameters may be interesting from an economic perspective. As mentioned above, the distinction between "free parameters" and "calibrated parameters" is largely conventional and depends on data availability as well as which parameters can be fixed on a priori grounds at "reasonable values"; see Dawkins et al. (2001). Further, the values of calibrated parameters can play a direct role in the conclusions drawn from a CGE analysis. For example, in Abdelkhalek and Dufour (1998), key parameters of the aggregate demand (Armington-type) function (such the share parameter between imported and domestic goods) appear as calibrated parameters. Similarly, in Abrego and Whalley (2000), calibrated parameters play a crucial role in decomposing wage inequality changes between contributing factors. So it appears it would be quite useful to have methods that can provide the investigator information on the accuracy and reliability of calibrated parameter estimates. But, essentially all the work on inference in CGE models has focused on the endogenous variables of the model, and is not directly applicable to calibrated parameters.

In this paper, we start by formalizing the calibration problem for CGE models, in a way that will allow the application of rigorous statistically-based methods (section 2). Two alternative forms of a CGE model are considered, which lead to different ways of specifying the way calibrated parameters appear in the model and how their uncertainty should be assessed. The first one retains a standard completely deterministic specification – like the vast majority of CGE models – while the second one gets closer to an econometric specification by allowing the introduction of random disturbances in the equations used for calibrating the model. In the first approach, calibrated parameters are functions of reference-period data as well as free parameters, and calibrated parameter uncertainty is induced by the free parameters. Drawing on the work of Abdelkhalek and Dufour (1998) on assessing endogenous variable uncertainty in CGE models, we then show how *projection* techniques can be used to build confidence regions for calibrated parameters (section 3). This enables the model builder to *account for the uncertainty associated with calibrated parameters* by constructing confidence regions for them using those of the free parameters. In sections 4 and 5, we illustrate this approach for the calibrated parameters of constant elasticity of substitution (CES) or transformation (CET) functions (the Armington form), frequently used in CGE models. In the second approach we consider (sections 6 to 8), we attempt to move beyond the deterministic framework typical of CGE models by allowing stochastic disturbances to appear in some of the equations. This induces calibrated parameter uncertainty even if the latter parameters do not depend on free parameters. We begin by discussing inference on the parameters of Cobb-Douglas type production functions (section 6) – a case where calibrated parameters are affected by disturbances but do not depend on free parameters – and we present an illustration based on a large disaggregate model of the Moroccan economy (section 7). The general case where calibrated parameters depend on both disturbances and free parameters is then discussed (section 8). We conclude in section 9.

## 2. Theoretical framework

In its most general form, a CGE model may be represented by a function $M$ such that

$$Y = M\left(X,\, \beta,\, \gamma\right) \tag{2.1}$$

where $Y$ is an $m$-dimensional vector of endogenous variables, $M$ is a (generally nonlinear) function which may be analytically quite complicated but remains computable, $X$ is a vector of exogenous or economic policy variables, $\beta$ is a $p$-dimensional vector of free parameters belonging to a subset $\Omega$ of $\mathbb{R}^p$, and $\gamma$ is a vector with $k$ elements containing the parameters to be calibrated. Note that the above equation obtains provided the model considered has a unique solution. Applied CGE models are typically constructed so that this condition holds; see Kehoe (1983), Mansur and Whalley (1984) and Dawkins et al. (2001).[1] Indeed, if the model did not have a solution, this would simply mean that the proposed structure is ill-conceived and should be modified. Similarly, multiple solutions indicate model incompleteness: since there is only one state of the world, some extra condition is needed to "pick" one. So again, this suggests that the model should be modified.

From a theoretical viewpoint, $\beta$ and $\gamma$ are not fundamentally different. However, they play different roles. While the elements of $\beta$ are parameters (*e.g.*, elasticities) of the behavioral equations of the model (utility/demand, production/supply, imports, exports, etc.), those of $\gamma$ are generally scale or share parameters. The calibration procedure consists in setting the vector of parameters $\gamma$ to *exactly* reproduce the data of a reference year, given a point estimate of the free parameters $\beta$ of the model. Thus, it is not surprising that the choice of these parameters has a large influence on the simulation results.

More formally, consider the equation:

$$Y_0 = M\left(X_0,\, \beta,\, \gamma\right) \tag{2.2}$$

where $Y_0$ and $X_0$ are vectors of endogenous and exogenous variables respectively for a given base

[1]The special form of the model from which (2.1) is derived does not matter for the methods described in this paper to apply. All we need is being able to solve the model. In particular, the latter may involve any functional form and market structure consistent with the existence of a solution, liquidity constraints, irreversibilities, "min"-type functions, production functions with complementary factors, rationing, etc.

year. Assuming that the solution for $\gamma$ exists and is unique, we can write:[2]

$$\gamma = H\left(Y_0,\, X_0,\, \beta\right) = h\left(\beta\right) \,. \tag{2.3}$$

When an estimate $\hat{\beta}$ of $\beta$ is available, the vector $\gamma$ is estimated by replacing $\beta$ with its estimate in equations (2.2) and (2.3).

Furthermore, we can usually decompose $\gamma$ into two subvectors $\gamma_1$ and $\gamma_2$, say $\gamma = (\gamma_1',\, \gamma_2')'$, where $\gamma_1$ (of dimension $k_1 \geq 0$) is independent of $\beta$. We can then write

$$\gamma_1 = h_1\left(Y_0,\, X_0\right) \,. \tag{2.4}$$

The second subvector $\gamma_2$ (of dimension $k_2 = k - k_1$) is, on the other hand, a function of $\beta$ as well as of $X_0$ and $Y_0$, hence

$$\gamma_2 = \bar{h}_2\left(Y_0,\, X_0,\, \beta\right) = h_2\left(\beta\right) \,. \tag{2.5}$$

To the extent that the vector of exogenous variables $(X)$ is known and taking into account the *deterministic* nature of the model, we can simplify the notation and write the model in the compact form

$$Y = \bar{g}\left(X,\, \beta\right) = g\left(\beta\right) \tag{2.6}$$

where the functions $\bar{g}$ and $g$ are defined for a given base year (after calibration), while $g$ also treats the vector $X$ as given. This formalization and qualifications on the calibrated parameters will prove to be very useful in theoretical developments and even indispensable for the numerical derivations associated with some approaches presented in this paper.

Generally, we will be interested in the effects of one or several economic policies which modify the elements of the vector $X$. Solutions to the model $M$, obtained for different values of exogenous variables $X$ but a single estimate value of $\beta$ may be compared and incorporated into a decision-making process. In principle, $\beta$ must be estimated econometrically, and it is possible to associate measures of uncertainty (standard deviations, confidence regions) with it. However, this type of information is generally ignored in appraisals of the reliability of the results.

We also note that the difficulties associated with the calibration of CGE models are not explicitly considered by usual methods for sensitivity analysis. These methods only deal with the estimation of the vector $\beta$, not $\gamma$. Notice that, in CGE models, the dimension of the joint vector $\left(\beta',\, \gamma'\right)'$ may be very large and econometric estimation difficult, if not impossible. In fact, the number of parameters of a CGE model increases rapidly with the number of sectors and consumers. Statistical data for high levels of disaggregation are frequently not available. The number of parameters to estimate may easily surpass the size of the sample. Thus, calibration may be viewed as an estimation procedure for $\gamma$; on this issue, see also Mansur and Whalley (1984, pp. 127–135) and Dawkins et al. (2001). It is clear that this procedure only yields point estimates and does not account for

[2]The assumption that the solution for $\gamma$ exists is, of course, essential to the possibility of calibrating the model. Otherwise, such methods cannot be used. Typical CGE model specifications, however, ensure that the solution exists and is unique; see Kehoe (1983), Mansur and Whalley (1984), Dawkins et al. (2001). On that issue, it is of interest to note that the methods proposed in this paper can in principle be applied even if multiple solutions can occur. We will discuss this possibility at the end of this section.

the uncertainty inherent in the estimation of the free parameters $\beta$, nor for that associated with the social-accounting matrix for the reference year [see Byron (1978)].

Even though typical specifications of CGE models have unique solutions and do allow a uniquely determined calibration, building CGE models is a complicated exercise and, in certain circumstances, we may end up with a model with multiple solutions. On that issue, it is of interest to note that the above setup can easily be modified to allow for multiple solutions of both the model and the calibration process, provided the set of possible solutions can be determined. In such a case, equation (2.1) is replaced by the set $\bar{Y}(X, \beta, \gamma)$ of all model solutions consistent with $(X, \beta, \gamma)$, *i.e.*

$$\bar{Y}(X, \beta, \gamma) = \{Y \in \mathbb{R}^m : \bar{M}(Y, X, \beta, \gamma) = 0\} \tag{2.7}$$

where $\bar{M}(Y, X, \beta, \gamma) = 0$ represents the restrictions imposed by the model, while $Y$ and $\gamma_2$ in (2.3) and (2.5) are replaced by the sets

$$\begin{aligned} \bar{\gamma}(Y_0, X_0, \beta) &= \{\gamma \in \mathbb{R}^k : Y_0 \in \bar{Y}(X_0, \beta, \gamma)\}, & (2.8)\\ \bar{\gamma}_2(Y_0, X_0, \beta) &= \{\gamma_2 \in \mathbb{R}^{k_2} : \gamma = (\gamma_1', \gamma_2')' \text{ and } Y_0 \in \bar{Y}(X_0, \beta, \gamma)\}. & (2.9)\end{aligned}$$

When there is unique model solution, the set $\bar{Y}(X, \beta, \gamma)$ contains only one point, in which case we can write:

$$\begin{aligned} \bar{\gamma}(Y_0, X_0, \beta) &= \{\gamma \in \mathbb{R}^k : Y_0 = M(X_0, \beta, \gamma)\}, & (2.10)\\ \bar{\gamma}_2(Y_0, X_0, \beta) &= \{\gamma_2 \in \mathbb{R}^{k_2} : \gamma = (\gamma_1', \gamma_2')' \text{ and } Y_0 = M(X_0, \beta, \gamma)\}. & (2.11)\end{aligned}$$

If the calibration process has a unique solution, these two sets reduce to single points. We will see in the next section that the projection method we suggest remains applicable to such solution sets (as opposed to unique solutions).

## 3. Projection-based confidence sets

In this section, we develop an approach that allows one to evaluate the uncertainty associated with the subvector of calibrated parameters, $\gamma_2$, deriving a confidence region from that of the vector of free parameters $\beta$. As in Abdelkhalek and Dufour (1998), we assume that we have a confidence region $C$ with level $1 - \alpha$ for the parameter $\beta$. In other words, $C$ is a subset of $\mathbb{R}^p$ such that

$$\mathsf{P}[\beta \in C] \geq 1 - \alpha \tag{3.1}$$

where $0 \leq \alpha < 1$. Two different interpretations may be put forward for the set $C$. First, we can assume that $C$ is a sampling (frequentist) confidence region based on previous statistical studies and observations, *i.e.* $C = C(Z)$ is a random subset of $\mathbb{R}^p$ generated by a sample $Z$ such that the probability that a given vector $\beta$ is contained within $C(Z)$ is greater than or equal to $1 - \alpha$. Second, in other situations we may treat the parameter $\beta$ as stochastic and consider that $\beta \in C$ is a Bayesian confidence region for $\beta$. It is important to note here *a complete prior distribution on the vector $\beta$ need not be specified*: all we require is the set $C$ and the probability $1 - \alpha$ assigned to it. For example, it is sufficient to have a set of prior (confidence) intervals on each element of $\beta$, whose joint probability is $1-\alpha$. We view this "partial information" feature as important: in practice, the investigator may find it relatively easy to assign a probability to an interval of possible values, but formulating a complete distribution on the parameter (especially, if is multidimensional) is a much more demanding exercise that may have unintended effects. So, in the sequel, we shall not assume that a complete distribution on $\beta$ is available (even though this is not precluded).[3] Similarly, if we use an estimator $\hat{\beta}$ to build the sampling confidence set (or interval) $C$, the distribution of the estimator need not be completely known: only what is needed to build the confidence set $C$ is required.

The arguments developed below are applicable under either of the above interpretations (frequentist or Bayesian). The region $C$ of $\mathbb{R}^p$ may be discrete, compact, connected or continuous. Under the assumption that (2.2) has a unique solution for $\gamma$, let $h_2(C)$ represent the image of $C$ over a calibration function $h_2$ defined in equation (2.5):

$$h_2(C) = \left\{\gamma_2 \in \mathbb{R}^{k_2} : \gamma_2 = h_2(\beta_0) \quad \text{for at least one } \beta_0 \in C\right\}. \tag{3.2}$$

If the assumption that the calibration process has a unique solution does not hold (provided a solution does exist), the image set can be defined in a more general way as follows:

$$h_2(C) = \left\{\gamma_2 \in \mathbb{R}^{k_2} : \gamma_2 \in \bar{\gamma}_2(Y_0,\, X_0,\, \beta) \quad \text{for at least one } \beta_0 \in C\right\} \tag{3.3}$$

where $\bar{\gamma}_2(Y_0,\, X_0,\, \beta)$ is given by (2.9). It is easy to see that the latter reduces to (3.2) when the solution of the calibration process is unique.

Clearly, we have the implication:

$$\beta \in C \Rightarrow h_2(\beta) \in h_2(C)\;, \tag{3.4}$$

hence

$$\mathsf{P}\left[\gamma_2 \in h_2(C)\right] \geq \mathsf{P}[\beta \in C] \geq 1-\alpha\,. \tag{3.5}$$

We see that $h_2(C)$ is a conservative confidence region for $\gamma_2$, with level greater than or equal to $1-\alpha$ [see Rao (1973, Section 7b.3, p. 473) or Gouriéroux and Monfort (1989, volume 2, pp. 243-250)].[4] In particular, when $C$ is a sampling confidence region for $\beta$, we have:

$$\mathsf{P}\left[h_2(\beta) \in h_2(C)\right] \geq \mathsf{P}[\beta \in C] \geq 1-\alpha\,, \quad \forall \beta \in \Omega\,. \tag{3.6}$$

We can also obtain individual confidence intervals for the elements $\gamma_{2i} = h_{2i}(\beta)$ of the vector $h_2(\beta) = (h_{21}(\beta),\, \ldots\,,\, h_{2k_2}(\beta))'$. In fact, since

$$h_2(\beta) \in h_2(C) \Rightarrow \left[h_{2i}(\beta) \in h_{2i}(C)\,,\ \text{for } i = 1,\, \ldots\,, k_2\right], \tag{3.7}$$

[3]Our partial information setup entails that the distribution of $\beta$ cannot be simulated because it is not completely specified. Similarly, a distribution for $\gamma$ is not available. On may wish to study what could be done once such extra assumptions are introduced, but this would go beyond the scope of the present paper.

[4]For further examples of the projection technique in econometrics, see Dufour (1989, 1990, 1997), Dufour and Kiviet (1996, 1998), Kiviet and Dufour (1997), and Dufour and Jasiak (2001).

we have:

$$\begin{aligned} \mathsf{P}\left[\gamma_{2j} \in h_{2j}(C)\right] &\geq \mathsf{P}\left[\gamma_{2i} \in h_{2i}(C),\ i = 1,\ \ldots,\ k_2\right] \\ &\geq \mathsf{P}\left[\gamma_2 \in h_2(C)\right] \\ &\geq 1 - \alpha,\ j = 1,\ \ldots,k_2. \end{aligned} \tag{3.8}$$

Since the function $h_2$ is generally nonlinear, the set $h_2(C)$ may be difficult to determine or visualize. In particular, it is not usually an interval or an ellipse. Nonetheless, as shown in Abdelkhalek and Dufour (1998), relatively simple forms may be derived from fairly weak assumptions on the function $h_2$ and on the set $C$ representing the confidence region of $\beta$.[5] In fact, if we assume that $h_2$ is continuous and that $C$ is compact in $\mathbb{R}^p$, the confidence region $h_2(C)$ for $\gamma_2$ is also compact in $\mathbb{R}^{k_2}$, and the univariate confidence regions for the elements of $\gamma_2$ are compact in $\mathbb{R}$. If $h_2$ is continuous and $C$ is connected in $\mathbb{R}^p$, the confidence region $h_2(C)$ for $\gamma_2$ is also connected in $\mathbb{R}^{k_2}$ and the confidence regions for the elements of $\gamma_2$ are connected in $\mathbb{R}$, and thus take the form of intervals. Finally, if $h_2$ is continuous and if $C$ is also *continuous* (*i.e.* connected, closed and bounded) in $\mathbb{R}^p$, then the confidence region $h_2(C)$ for $\gamma_2$ is also continuous in $\mathbb{R}^{k_2}$, and the univariate confidence intervals are continuous in $\mathbb{R}$. In particular, in this case the individual confidence regions $h_{2i}(C),\ i = 1,\ \ldots,\ k_2$, take the shape of closed and bounded intervals: $h_{2i}(C) = \left[\gamma_{2i}^L,\ \gamma_{2i}^U\right]$, where $\gamma_{2i}^L > -\infty$ and $\gamma_{2i}^U < +\infty,\ i = 1,\ \ldots,\ k_2$.

In general, we can always construct simultaneous confidence intervals for the different elements of $h_2(\beta)$. We simply consider the extreme values:

$$h_{2i}^L(C) = \inf\{h_{2i}(\beta) : \beta \in C\},\ h_{2i}^U(C) = \sup\{h_{2i}(\beta) : \beta \in C\} \tag{3.9}$$

where $-\infty \leq \gamma_{2i}^L < \infty$ and $-\infty < \gamma_{2i}^U \leq \infty,\ i = 1,\ \ldots,\ k_2$. Since $[h_{2i}(\beta) \in h_{2i}(C),\ i = 1,\ \ldots,\ k_2] \Rightarrow [h_{2i}^L(C) \leq h_{2i}(\beta) \leq h_{2i}^U(C),\ i = 1, \ldots k_2]$, we have:

$$\begin{aligned} \mathsf{P}\left[h_{2j}^L(C) \leq \gamma_{2j} \leq h_{2j}^U(C)\right] &\geq \mathsf{P}\left[h_{2i}^L(C) \leq \gamma_{2i} \leq h_{2i}^U(C),\ i = 1,\ \ldots,\ k_2\right] \\ &\geq \mathsf{P}\left[\gamma_{2i} \in h_{2i}(C),\ i = 1,\ \ldots,\ k_2\right] \\ &\geq 1 - \alpha,\ \text{for } j = 1,\ \ldots,\ k_2. \end{aligned} \tag{3.10}$$

It is thus sufficient to minimize and maximize each element of $\gamma_2 = h_2(\beta)$ subject to the constraint $\beta \in C$ to obtain (simultaneous) level $1 - \alpha$ confidence intervals for all of them.

Using these results we can construct confidence regions for the endogenous variables of CGE models from the confidence regions of the two parameter vectors $\beta$ and $\gamma_2$ (free, and calibrated dependent on the free) or simply from those of the vector $\beta$ of free parameters, having eliminated the calibrated parameters depending on the free parameters, while accounting for the uncertainty associated with them. This result allows us to substantially simplify the numerical procedures, especially when the dimension of vector $\gamma_2$ is large. We illustrate the process of building these confidence regions of type $\gamma_2(C)$ with an example in the following sections.

[5] These assumptions are satisfied by the functional forms typically used in CGE models.

# 4. Calibrated parameters for CES and CET functions

To illustrate the approach proposed above, we will now perform a detailed analysis of the case of an *Armington-type* import function commonly used in CGE models. This general *constant elasticity of substitution (CES)* form, which can be subject to various interpretations, is used to model sectorial production, exports, portfolio composition (models with financial flows), etc. In other words, this example covers a large number of cases of calibration in the presence of free parameters (elasticities) in CGE models. This function is linearly homogenous in its arguments, the number of which depends upon the model (inputs or factors of production, origin of imports, markets for exports, substitutable financial assets). In our example we consider an import model in which a consumer derives utility from consuming a composite good denoted $Q$. This good is comprised of imported goods $M$ and domestic goods $D$. The consumer's problem is to choose a combination of quantities $M$ and $D$ which minimizes overall expenditure, given the two prices $p_M$ and $p_D$ and the level $Q$. The Armington form of this CES function is given by

$$Q = B\left[\delta M^{-\rho} + (1-\delta)\, D^{-\rho}\right]^{-1/\rho}. \tag{4.1}$$

To find a more direct interpretation, we let $\sigma = 1/(1+\rho)$, *i.e.* $\rho = (1-\sigma)/\sigma$. Equation $(4.1)$ may then be rewritten:

$$Q = B\left[\delta M^{(\sigma-1)/\sigma} + (1-\delta)\, D^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \tag{4.2}$$

where $B$ is a constant, $\delta$ a share parameter, and $\sigma$ a (constant) elasticity of substitution between imported and domestic goods. In our terminology, given the deterministic calibration procedures applied to this type of function in CGE models [see Mansur and Whalley (1984)], $B$ and $\delta$ are *calibrated* parameters while $\sigma$ (or $\rho$) is a *free* parameter estimated or borrowed from outside the model, independent of the data from the social-accounting matrix for the reference year. The first-order condition associated with this problem is given by the equality between the price ratio for the two types of good and the marginal rate of substitution between imported and domestic goods:

$$\frac{p_D}{p_M} = \left[\frac{(1-\delta)}{\delta}\right]\left(\frac{D}{M}\right)^{-\rho-1} \tag{4.3}$$

or

$$\frac{M}{D} = \left[\frac{\delta p_D}{(1-\delta)\, p_M}\right]^{1/(\rho+1)} = \left(\frac{\delta}{1-\delta}\right)^{\sigma}\left(\frac{p_D}{p_M}\right)^{\sigma}. \tag{4.4}$$

This method of modelling imports, examined in detail by de Melo and Robinson (1989) and by Devarajan, Lewis and Robinson (1990), is extensively used in CGE models.[6] This seems more realistic than the classic formulation with perfect substitutability between goods. The CES function is sufficiently tractable for the analytical derivations and the calibration of parameters, despite the fact that it introduces a free parameter.

To calibrate the parameters of this type of function in CGE models, different techniques have

[6] For a review of general equilibrium studies having used these forms, see Decaluwé and Martens (1988).

been used (estimates, literature reviews, international comparisons, or arbitrary fixing) to assign a value $(\hat{\sigma})$ to the free parameter — the elasticity of substitution $(\sigma)$ in this case. This value is crucial and constitutes the first step of the calibration process. From the first order condition [equation (4.4)], from the data for $Q_0$, $M_0$ and $D_0$, and from a normalization assumption imposed on the base-year prices, we derive:[7]

$$\left(\frac{M_0}{D_0}\right)^{1/\hat{\sigma}} = \frac{\delta}{1-\delta}\left(\frac{p_{D_0}}{p_{M_0}}\right) \tag{4.5}$$

yielding a *unique* estimate for $\delta$ given by

$$\hat{\delta} = \frac{\frac{p_{M_0}}{p_{D_0}}\left(\frac{M_0}{D_0}\right)^{1/\hat{\sigma}}}{1+\frac{p_{M_0}}{p_{D_0}}\left(\frac{M_0}{D_0}\right)^{1/\hat{\sigma}}} = h_{21}\left(\hat{\sigma}\right) \; . \tag{4.6}$$

Now it remains to calibrate the scale parameter $B$. From equation $(4.2)$ and from the base-year data, we find:

$$\hat{B} = Q_0/\left[\,\hat{\delta}M_0^{(\hat{\sigma}-1)/\hat{\sigma}} + \left(1-\hat{\delta}\right)D_0^{(\hat{\sigma}-1)/\hat{\sigma}}\,\right]^{\hat{\sigma}/(\hat{\sigma}-1)} = \; h_{22}\left(\hat{\sigma}\right) \; . \tag{4.7}$$

In equations (4.6) and (4.7) the essential role played by the free parameter in determining the values of the other parameters appears clearly. From this deterministic approach to calibration, we seek to construct confidence intervals for the two calibrated parameters, $\delta$ and $B$, given that of the free parameter $\sigma$. To achieve this, we work not with a point estimate for $\sigma$, $\hat{\sigma}$, but rather with a set estimate.

Moving from the definition for a continuous function of a confidence interval $C \subset \mathbb{R}$ for $\sigma$ given in expression (4.6) towards a subset $h_{21}\left(C\right) \subset \mathbb{R}$, we analytically illustrate the construction of what is to become a confidence interval for $\delta$. To simplify notation, we write:

$$N\left(\sigma\right) = \frac{p_{M_0}}{p_{D_0}}\left(\frac{M_0}{D_0}\right)^{1/\sigma} = \left(\frac{p_{M_0}}{p_{D_0}}\right)e^{\frac{1}{\sigma}\ln\left(\frac{M_0}{D_0}\right)} \; , \tag{4.8}$$

hence

$$\delta = h_{21}\left(\sigma\right) = \frac{N\left(\sigma\right)}{1+N\left(\sigma\right)} \; . \tag{4.9}$$

We now wish to examine the behavior of the function $h_{21}$, particularly within the confidence interval $C$. From equation (4.9) we see that

$$\frac{d\delta}{d\sigma} = \frac{N'\left(\sigma\right)\left[1+N\left(\sigma\right)\right] - N'\left(\sigma\right)N\left(\sigma\right)}{\left[1+N\left(\sigma\right)\right]^2} = \frac{N'\left(\sigma\right)}{\left[1+N\left(\sigma\right)\right]^2} \tag{4.10}$$

[7] An assumption concerning the base-year prices is usually made in CGE models. All prices, except those which include taxes or subsidies, are normalized to one for the base year (*i.e.* are treated as indices).

where

$$N'(\sigma) = \frac{dN}{d\sigma} = \frac{p_{M_0}}{p_{D_0}} \left[ -\frac{1}{\sigma^2} \ln \left( \frac{M_0}{D_0} \right) \right] \left( \frac{M_0}{D_0} \right)^{1/\sigma} . \tag{4.11}$$

So it is clear that the sign of $d\delta/d\sigma$ is the same as that of $dN/d\sigma$, *i.e.*

$$sgn \left( \frac{d\delta}{d\sigma} \right) = sgn \left( \frac{dN}{d\sigma} \right) = sgn \left[ -\ln \left( \frac{M_0}{D_0} \right) \right] = sgn \left[ \ln \left( \frac{D_0}{M_0} \right) \right] \tag{4.12}$$

where $sgn(x) = 1$ if $x > 0$, $sgn(x) = -1$ if $x < 0$, and $sgn(x) = 0$ if $x = 0$. If $D_0 > M_0$, then $(d\delta/d\sigma) > 0$ and vice versa. This result, which we have never encountered in the literature on CGE models, is quite surprising and, depending on the context, it may have interesting economic interpretations. We see that the function $h_{21}$ is continuous and strictly monotonic. If we assume that the confidence interval for $\sigma\,(C)$ is a closed bounded set of the form $[\underline{\sigma}, \overline{\sigma}]$, with level $1 - \alpha$, then one of the two intervals $[h_{21}(\underline{\sigma}), h_{21}(\overline{\sigma})]$ and $[h_{21}(\overline{\sigma}), h_{21}(\underline{\sigma})]$ is a level $1-\alpha$ confidence interval for the calibrated parameter $\delta$.[8] In other words, one of the following implications must hold:

$$\mathsf{P}(\sigma \in [\underline{\sigma}, \overline{\sigma}]) \geq 1 - \alpha \Rightarrow \mathsf{P}(\delta \in [h_{21}(\underline{\sigma}), h_{21}(\overline{\sigma})]) \geq 1 - \alpha , \tag{4.13}$$

$$\mathsf{P}(\sigma \in [\underline{\sigma}, \overline{\sigma}]) \geq 1 - \alpha \Rightarrow \mathsf{P}(\delta \in [h_{21}(\overline{\sigma}), h_{21}(\underline{\sigma})]) \geq 1 - \alpha . \tag{4.14}$$

In addition to the share parameter $\delta$, a similar analysis may be performed on the scale parameter $B$. This work is analytically not as simple as that on $\delta$, but remains feasible numerically (see section 5).

Since the calibration procedure is usually performed in a pre-defined order, accounting for the uncertainty associated with the calibrated parameters depending on the free parameters is tantamount to clearly specifying the confidence regions for the free parameters of the model.

## 5. Application to aggregate CGE models of Morocco

In this section, we apply the projection method described in section 3 to the construction of confidence sets for the calibrated parameters (which depend on free parameters) in the context of two different models for Morocco. The first one is a submodel of a type 1-2-3 CGE model [Devarajan et al. (1990)] studied in Abdelkhalek and Dufour (1998). The second one is a submodel of a two-sector model (agriculture and industry) used by Abdelkhalek and Martens (1996). Both models include imported goods $(M)$ and locally produced goods $(D)$, which are aggregated through an Armington-type CES function. The Moroccan reference year data come from 1985 for the first model and from 1990 for the second model. Calculations and optimizations were performed using the GAMS-MINOS program [see Brooke, Kendrick and Meeraus (1988)].[9]

Given the reference-year values $Q_0$, $M_0$, $D_0$, $p_{M_0}$ and $p_{D_0}$ and a level $1 - \alpha$ confidence region $C$ for the free parameter $\sigma$, the confidence intervals are obtained by minimizing and maximizing the

[8] A study by Reinert and Roland-Holst (1992) of 163 sectors of the U.S. economy reveals that this elasticity $\sigma$ falls between 0.14 and 3.49.

[9] The program is supplied in Appendix B.

*functions* which define the calibrated parameters subject to the restriction that the free parameter remains in its confidence region. Note the confidence set $C$ for $\sigma$ may be truncated to only contain values in a set $C_0$ of economically admissible values; the resulting smaller confidence set $C \cap C_0$ has the same level as the original set $C$ [see Abdelkhalek and Dufour (1998)]. The set $C \cap C_0$ to which $\sigma$ is restricted is usually specified through a set of nonlinear inequalities. More precisely, we solve the following problems :

$$\begin{aligned} \text{minimize and maximize } \delta &= h_{21}(\sigma) \equiv \frac{(p_{M_0}/p_{D_0})\,(M_0/D_0)^{1/\sigma}}{1+(p_{M_0}/p_{D_0})\,(M_0/D_0)^{1/\sigma}}\,, \\ \text{subject to } \sigma &\in C \cap C_0\,; \end{aligned} \tag{5.1}$$

$$\begin{aligned} \text{minimize and maximize } B &= h_{22}(\sigma) = Q_0/\left[\,\delta M_0^{(\sigma-1)/\sigma} + (1-\delta)\,D_0^{(\sigma-1)/\sigma}\,\right]^{\sigma/(\sigma-1)} \\ \text{subject to } \delta &= h_{21}(\sigma) \quad \text{and} \quad \sigma \in C \cap C_0\,. \end{aligned} \tag{5.2}$$

It is also useful to remember that the price of the imported good is given by the equation

$$p_{M_0} = p_{wm0}(1+t_m)E_0 \tag{5.3}$$

where $p_{wm0}$ is the international price of imports, $t_m$ is the tariff on imports and $E_0$ is the nominal exchange rate, evaluated at the reference year.

The Moroccan data used in our calculations are summarized in Table 1, while the confidence intervals for calibrated parameters $\delta$ and $B$ appear in Table 2. For the one-sector model calibrated on the reference year 1985, we used for the free parameter $\sigma$ the 95% confidence interval $[0.7838, 2.0809]$, which is based on the estimations presented in Abdelkhalek and Dufour (1998). The results in Table 2 indicate that this interval on $\sigma$ gets translated into the intervals $[0.137, 0.361]$ and $[1.568, 1.862]$ for $\delta$ and $B$ respectively. These intervals show there is a non-negligible uncertainty on the calibrated parameters even though the confidence intervals remain remarkably tight and informative. For the two-sector model (calibrated on 1990 data), we used the wider interval $[0.5, 4.5]$. The latter was a subjectively determined, although quite consistent with the range of values reported by Reinert and Roland-Holst (1992) for similar elasticities. Not surprisingly, we find in this case wider (although still informative) intervals for the sectorial parameters $\delta$ and $B$ associated with agriculture and industry: $\delta \in [0.004, 0.331]$ and $B \in [1.010, 1.658]$ for the agricultural sector, $\delta \in [0.058, 0.458]$ and $B \in [1.470, 1.988]$ for the industrial sector.

## 6. Confidence regions based on equations with disturbances

In this section, we present an approach for constructing confidence regions for the calibrated parameters of the model, which goes beyond the deterministic calibration process which is typical of CGE models. This method introduces randomness, and thus uncertainty, into some of the model equations – namely those which are used in the calibration process – used for the deterministic

TABLE 1: Moroccan data used in the calibrations[a]

| Variables | SAM 1985 | SAM 1990 Agriculture | SAM 1990 Industry |
|---|---|---|---|
| $Q_0$ | 252653 | 69589.32 | 317195.92 |
| $M_0$ | 42806 | 4248 | 59327.9 |
| $D_0$ | 209847 | 65341.32 | 257868.02 |
| $TAXM_0$ | 9046.7 | -391.79 | 10048.1 |
| $tm$ | 0.211 | -0.0922 | 0.16936 |
| $PD_0$ | 1 | 1 | 1 |
| $PWM_0$ | 1 | 1 | 1 |
| $E_0$ | 1 | 1 | 1 |
| $PM_0$ | 1.211 | 0.9078 | 1.16936 |
| $\sigma$ | [0.7838, 2.0809] | [0.5, 4.5] | [0.5, 4.5] |

[a]SAM: social accounting matrix. Data for $Q_0$, $M_0$, $D_0$, $TAXM_0$ are in millions of dirhams and were obtained from Groupe de recherche en économie internationale (G.R.E.I.) (1992) for 1985 and from Abdelkhalek and Martens (1996) for 1990. The confidence intervals for $\sigma$ are econometric estimates for 1985 from Abdelkhalek and Dufour (1998), while those for 1990 are subjectively determined although consistent with the elasticity values reported by Reinert and Roland-Holst (1992).

TABLE 2: Confidence intervals for $\delta$ and $B$

| Parameter | $\delta$ | | | $B$ | | |
|---|---|---|---|---|---|---|
| Confidence bounds | Lower | Upper | Range | Lower | Upper | Range |
| 1985 | 0.137 | 0.361 | 0.224 | 1.568 | 1.862 | 0.294 |
| Agriculture 1990 | 0.004 | 0.331 | 0.327 | 1.010 | 1.658 | 0.648 |
| Industry 1990 | 0.058 | 0.458 | 0.400 | 1.470 | 1.988 | 0.518 |

calibration, in order to construct simultaneous confidence regions for the calibrated parameters. Random disturbances will be introduced in a way similar to what is usually done routinely in econometric models, in view of allowing for "perturbations" to equilibrium conditions, associated with measurement errors or approximation errors. In particular, we will allow for perturbations only in the equations used in the calibration exercise, an important simplifying feature. The method can also be applied to study the reliability of calibrated parameters even if free parameters do not appear in the equations considered.

The "random errors" we consider here are not meant to transform a fundamentally non-stochastic model into a stochastic model where agents would maximize expected utility taking account a stochastic environment. The non-stochastic nature of the underlying behavioral is not modified (although this would certainly be of interest). The exercise we perform here is meant to be a first step in the direction of allowing for the presence of random disturbances in the model. [10]

Before generalizing the proposed approach (section 8), we shall discuss the important case of a Cobb-Douglas production function with constant returns to scale for the factors labor and capital. Since primary inputs are required in the production process, "production" is defined as value added. This type of modelling and these functional forms are frequently used in CGE models because of the simplicity, of the resulting expressions and calibration. The general form of this type of production function in the presence of several categories of the labor input and a single factor capital per sector is given by:

$$X_i = A_i \prod_l L_{i,l}^{\delta_{i,l}} K_i^{\left(1-\sum_l \delta_{i,l}\right)} . \tag{6.1}$$

where $X_i$ is production (or value added) in sector $i$, $A_i$ is a scale parameter, $L_{i,l}$ is the quantity of the type $l$ labor used in sector $i$, $K_i$ is the quantity of capital used in sector $i$, and $\delta_{i,l}$ the elasticity of production of type $l$ labor in sector $i$. All of the following presentation may be derived from equation (6.1). In order to simplify the notation, we shall ignore the index $i$ representing the sector and consider only one type of labor. Thus, production function (6.1) assumes the following simpler form:

$$X = AL^{\delta} K^{(1-\delta)} . \tag{6.2}$$

In standard CGE models certain assumptions are made concerning the structure of markets. These assumptions facilitate accounting for the behavior of agents, particularly of firms, in each sector of the economy. This information is used to derive factor demands from profit maximization programs. Since our concern here is primarily econometric, we shall assume that the sector is perfectly competitive. One of the first-order conditions is:

$$pX\delta = wL, \tag{6.3}$$

where $p$ is the price of good $X$ (or the price of the value added), and $w$ is the wage rate of labor. To calibrate the parameters of this type of function, model builders only require reference data for the base year from a social-accounting matrix. No information on the free parameters is required.

[10] As we will see, even this simple first step, introduces non-negligible extra complications. A potentially useful extra step would consist in considering a fully stochastic behavioural model, but this would require important modifications to our basic setup and would go beyond the purpose of this paper.

The necessary data for the reference year comprise: the sectorial value of production (or the value added, $p_0 X_0$) and the corresponding total wage bill $(w_0 L_0)$. Equation (6.3) then yields a *unique* estimate for $\delta$, based on a single observation (the base year), owing to the deterministic nature of the model:

$$\hat{\delta} = \frac{w_0 L_0}{p_0 X_0}. \tag{6.4}$$

From this estimate for $\delta$ and from equation (6.2), we can derive an estimate for the scale coefficient of the production function $(A)$ :[11]

$$\hat{A} = \frac{X_0}{L_0^{\hat{\delta}} K_0^{\left(1-\hat{\delta}\right)}}. \tag{6.5}$$

Given the aforementioned assumptions, it is obvious that this calibration procedure may be applied to production functions of this type for all sectors and factors of production.

We now consider a production function (6.2) incorporating a stochastic disturbance term — applicable to this example as well as similar ones. The notion of introducing random shocks into some of the equations of a CGE model (those used in the deterministic calibration) is not entirely new. Mansur and Whalley (1984) proposed stochastic forms for CGE models which allow the estimation of the parameters, provided there is a sufficient number of observations. However, this is generally not possible (*e.g.,* when the data only pertains to a single base year). This is the case we are concerned with here.

The random perturbations introduced have the same interpretation as the error terms in econometric models: they represent the effect of observation errors, specification errors, random deviations from optimal behavior among economic agents, etc. Of course, introducing such perturbations requires one to specify the error distribution and complicates the model.

Assume that the production function (6.2) and the first-order condition (6.3) are stochastic, as follows:

$$X = AL^{\delta} K^{(1-\delta)} e^{u}, \tag{6.6}$$

$$pX\delta = wLe^{v}, \tag{6.7}$$

where $(u,\, v)$ is a vector of random variables with a known distribution that can be simulated. Here the error term $u$ in the first equation is a (fairly standard) perturbation of the production function, while $v$ may capture a temporary deviation from first-order profit maximization condition due to market imperfections, measurement errors, agent errors, etc.

Two equations can be written for the base-year data:

$$X_0 = AL_0^{\delta} K_0^{(1-\delta)} e^{u_0}, \tag{6.8}$$

$$p_0 X_0 \delta = w_0 L_0 e^{v_0}, \tag{6.9}$$

[11] Notice that we have again normalized the prices, allowing us to impute the value $p_0 X_0$ to the volume $X_0$.

where $A$ and $\delta$ are the two unknown parameters. We then deduce:

$$\delta = \frac{w_0 L_0}{p_0 X_0} e^{v_0} , \tag{6.10}$$

$$A = \frac{X_0}{L_0^{\delta} K_0^{(1-\delta)} e^{u_0}} . \tag{6.11}$$

The equations for the deterministic framework derived in (6.4) and (6.5) no longer obtain, because they only hold true when the random errors $u$ and $v$ are identically zero. In the stochastic model these two equations yield estimators for $\delta$ and $A$ respectively. Thus, by definition,

$$\hat{\delta}_0 = \frac{w_0 L_0}{p_0 X_0} \tag{6.12}$$

irrespective of $A$. Let

$$A_0 \equiv A_0 (\delta) = \frac{X_0}{L_0^{\delta} K_0^{(1-\delta)}} . \tag{6.13}$$

In particular, for $\hat{\delta}_0$ we obtain:

$$\hat{A}_0 = \hat{A}_0 \left(\hat{\delta}_0\right) = \frac{X_0}{L_0^{\hat{\delta}_0} K_0^{\left(1-\hat{\delta}_0\right)}} . \tag{6.14}$$

From equations (6.11) and (6.14) we find

$$\frac{\hat{A}_0}{A} = \frac{L_0^{\delta} K_0^{(1-\delta)} e^{u_0}}{L_0^{\hat{\delta}_0} K_0^{\left(1-\hat{\delta}_o\right)}} \tag{6.15}$$

or, equivalently,

$$\frac{\hat{A}_0}{A} = \left(\frac{K_0}{L_0}\right)^{(\hat{\delta}_0-\delta)} e^{u_0} \tag{6.16}$$

which, upon taking logs, yields

$$\ln\left(\hat{A}_0\right) - \ln(A) = \left(\hat{\delta}_0 - \delta\right) \ln\left(K_0/L_0\right) + u_0 . \tag{6.17}$$

Furthermore, from equations (6.10) and (6.12) we derive:

$$\frac{\hat{\delta}_0}{\delta} = \frac{1}{e^{v_0}} \tag{6.18}$$

or

$$\ln(\delta) - \ln\left(\hat{\delta}_0\right) = v_0 . \tag{6.19}$$

From either equation (6.18) or (6.19), we see that $\hat{\delta}_0/\delta$ or $\ln\left(\hat{\delta}_0\right) - \ln(\delta)$ are *pivotal* functions for the parameter $\delta$. A pivotal function for $\delta$ is any stochastic function $Z$ defined on the observations and on the parameter $\delta$ such that the distribution of $Z$ does not depend on $\delta$ despite the fact that this parameter appears in the arguments [see Gouriéroux and Monfort (1989, volume 2, p. 24), for example]. When we have a pivotal function which can be inverted to isolate the parameter of interest, we can construct confidence intervals for that parameter. This is the procedure we shall use here. Given any known distribution of the vector $(u_0,\, v_0)$ , simulated confidence intervals may be constructed for the parameters $\delta$ and $A$ or for functions of these parameters. Notice that, unlike Mansur and Whalley (1984), we require neither that $(u_0,\, v_0)$ be normally distributed nor that $u_0$ and $v_0$ be independent. However, by making these assumptions we benefit from significant practical simplifications.

In the case we are about to examine, notice that $\hat{\delta}_0/\delta$ only depends upon $v_0$ (and not on $u_0$). We can write

$$\mathsf{P}\big[\,\hat{\delta}_0/\delta > c_\alpha\big] = \mathsf{P}\big[\,e^{-v_0} > c_\alpha\big] = \alpha \tag{6.20}$$

where $\alpha$ is a constant fixed a priori and $c_\alpha$ is the corresponding critical value, which can be derived from the theoretical or simulated distribution of $v_0$. Thus we have

$$\mathsf{P}\big[\,\hat{\delta}_0/\delta \le c_\alpha\big] = 1-\alpha\,, \tag{6.21}$$

and

$$\mathit{\Gamma}_\delta = \big\{\delta\in\mathbb{R} : \hat{\delta}_0/\delta \le c_\alpha\big\} = \left\{\delta\in\mathbb{R} : c_\alpha\delta \ge \frac{w_0 L_0}{p_0 X_0}\right\} \tag{6.22}$$

is a level $1-\alpha$ confidence interval for the parameter $\delta$.

Similarly, we can construct a confidence interval for the parameter $A$. In a first instance, if we assume that $\delta$ is known and that the unknown parameter is the scale parameter $A$, we can use equations (6.11) and (6.13) to derive:

$$e^{u_0} = A_0/A \tag{6.23}$$

which, as before, yields a pivotal function for $A$ and allows the construction of a confidence interval for this parameter. Nonetheless, as $\delta$ is generally unknown, this procedure may not be very useful. The two equations (6.16) and (6.17) cannot yield a pivotal function for $A$. Even if we use equations (6.17) and (6.18) to eliminate $\delta$, the ensuing expression

$$u_0 = \ln\big(\hat{A}_0\big) - \ln(A) + \hat{\delta}_0\,(e^{v_0}-1)\ln(K_0/L_0) \tag{6.24}$$

does not constitute a pivotal function for $A$.

When $\delta$ is unknown it is thus difficult (if not impossible) to construct a similar confidence interval for $A$. Nonetheless, it is possible to find a two-dimensional pivotal function for the two-dimensional parameter $(A,\delta)$ . Using equations (6.17) and (6.19) we may write:

$$W = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} = \begin{pmatrix} \ln\big(\hat{A}_0\big) - \ln(A) + \big(\delta-\hat{\delta}_0\big)\ln(K_0/L_0) \\ \ln(\delta) - \ln\big(\hat{\delta}_0\big) \end{pmatrix}. \tag{6.25}$$

Since the distribution of the vector $(u_0, v_0)$ is fixed and known by assumption, we indeed have a pivotal function for the pair $[\ln(A)\,, \ln\delta]$ . Since the covariance matrix, $\mathit{\Omega}$, of this vector is known, we can calculate the following statistic:

$$T(u_0, v_0) = W' \Omega^{-1} W\,. \tag{6.26}$$

By assumption, this distribution can be simulated. In particular, if we assume that the distribution of vector $(u_0, v_0)$ is multivariate normal, this distribution will be $\chi^2(2)$. Consequently, we can find the point $c_\alpha$ such that

$$\mathsf{P}\left[T(u_0, v_0) \leq c_\alpha\right] = \mathsf{P}\left[W' \Omega^{-1} W \leq c_\alpha\right] = 1 - \alpha\,, \tag{6.27}$$

where $\alpha$ is a level fixed a priori. Finally, a level $1-\alpha$ confidence region for the pair $(A, \delta)$ is given by

$$\Gamma_{(\delta, A)} = \left\{(\delta,\, A) \in \mathbb{R}^2 : W' \Omega^{-1} W \leq c_\alpha\right\}\,. \tag{6.28}$$

Once the simultaneous confidence set has been obtained, we can build individual confidence intervals for $\delta$ and $A$ by projection, which can be done by finding the minimal and maximal values $\delta$ and $A$ over the set $\Gamma_{(\delta,\, A)}$.

The procedure described in this section for Cobb-Douglas production functions covers a number of cases used in CGE models. Similar cases may be dealt with using the same techniques to construct confidence regions for all the calibrated parameters of a model.

# 7. Application to a disaggregate model of the Moroccan economy

We have applied the method described in section 6 to a fairly large model of the Moroccan economy aimed at studying the effect of trade liberalization on poverty [Abdelkhalek (2003)].[12] The model has 34 production sectors whose value-added (VA) are determined by two-factor Cobb-Douglas production functions, where the parameters (the scale parameter $A$ and the elasticity $\delta$) can differ across sectors. There is no econometric information on these important parameters, so they were determined by calibration, using the 1998 input-output table of the Moroccan economy. Further, it is of interest to have an idea how reliable these calibration-based estimates are.

The calibrated parameters were obtained using the equations (6.4) - (6.5). In order to evaluate the sensitivity of these results to "perturbations" of the equations, we considered the modified equations (6.8) - (6.9). For that purpose, we shall assume that the errors $u_0$ and $v_0$ are normal with mean zero and variance $\sigma_0^2$. The assumption on the value of $\sigma_0$ controls the tightness of the intervals. Standard non-stochastic calibration corresponds to the special case where $\sigma_0 = 0$. For this exercise, we considered the values $\sigma_0 = 0.25$ and $0.5$. Given the normal range of the variables involved, these standard errors are quite "large" (especially for $0.5$). Values below $0.25$ are clearly the most reasonable. Of course, other distributions could be considered: the present example is mainly meant to be illustrative.[13]

[12] This study was supported by a World Bank contract.

[13] We should also note here that the normality assumption is strictly speaking incompatible with the fact the VA share of any sector with respect to any total value added ($s_0 \equiv w_0 L_0 / p_0 X_0$) must be smaller than one [negative values are precluded by equation (6.9)]. In principle, the distribution of $v_0$ should be truncated so that $s_0$ cannot be larger than one. Given the values of $\sigma_0$ used, the probability of getting an value of $s_0$ larher than one is essentially zero, so that results

TABLE 3: Confidence intervals for $\delta$ and $B$ with stochastically disturbed equations (0.95 level)

| Sector | VA share | $\delta$ | | | | $A$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Point est. | Lower bound | Upper bound | Range | Point est. | Lower bound | Upper bound | Range |
| $\sigma_0 = 0.25$ | | | | | | | | | |
| AGRI | 17.63 | 0.0272 | 0.0166 | 0.0444 | 0.0278 | 1.1329 | 0.6940 | 1.8492 | 1.1552 |
| COREP | 14.12 | 0.1782 | 0.1092 | 0.2908 | 0.1816 | 1.5978 | 0.9788 | 2.6081 | 1.6293 |
| IMLSRE | 11.38 | 0.0816 | 0.05 | 0.1331 | 0.0831 | 1.3265 | 0.8127 | 2.1653 | 1.3526 |
| $\sigma_0 = 0.50$ | | | | | | | | | |
| AGRI | | | 0.0102 | 0.0724 | 0.0622 | | 0.4252 | 3.0186 | 2.5934 |
| COREP | | | 0.0669 | 0.4747 | 0.4078 | | 0.5997 | 4.2572 | 3.6575 |
| IMLSRE | | | 0.0306 | 0.2173 | 0.1867 | | 0.4979 | 3.5345 | 3.0366 |

We report in Table 3 the results of this exercise for three most sectors (in terms of share of total value added) in the model: agriculture, forestry and fishing (AGRI), trade and repair services (COREP) and real estate renting and services to forms (IMLSRE). We consider again intervals with level 0.95. We see from these results that the intervals for $\delta$ are reasonably tight with $\sigma_0 = 0.25$, although appreciably wider with $\sigma_0 = 0.5$. Further details are available in Abdelkhalek (2003).

## 8. Confidence regions from equations with disturbances: general approach

In this section, we generalize the approach based on simulations to construct confidence regions, compatible with an underlying *deterministic* calibration, for all the calibrated parameters of a CGE model. To accomplish this, we revert to the first three equations describing the basic structure of the problem, given in section 2:

$$
\begin{aligned}
Y &= M(X,\, \beta,\, \gamma)\ , && (8.1)\\
Y_0 &= M(X_0,\, \beta,\, \gamma)\ , && (8.2)\\
\gamma &= H(Y_0,\, X_0,\, \beta) = h(\beta)\ . && (8.3)
\end{aligned}
$$

To begin, assume that there are no free parameters in the model, *i.e.* that all parameter values can be derived from the reference-year data (such as a social-accounting matrix). We shall return to examine the case with free parameters. Equations (8.1), (8.2) and (8.3) can thus be simplified to:

$$
Y = \bar{M}(X,\, \gamma)\ , \tag{8.4}
$$

based on a truncated normal distribution would be the same for all prectical purposes.

$$
\begin{aligned}
Y_0 &= \bar{M}\left(X_0,\, \gamma\right), && (8.5)\\
\gamma &= \bar{H}\left(Y_0,\, X_0\right). && (8.6)
\end{aligned}
$$

Contrary to what is suggested by the general formulation of the model as expressed above, the calibration process usually only uses *some of the equations* of the model. Generally, these are the equations which specify the behavior of agents, the corresponding first-order conditions, and sometimes certain equilibrium conditions. The remaining equilibrium conditions, the accounting identities, and the definitions are not used in the calibration. For this reason, and because it is the econometric aspect of calibration that we are interested in, we may rewrite the calibration sub-system as

$$
Y^S = S\big(X^S,\, \gamma\big)\,, \tag{8.7}
$$

hence

$$
\gamma = \bar{H}_S\big(Y_0^S,\, X_0^S\big) \tag{8.8}
$$

where $Y_0^S$ and $X_0^S$ respectively represent the model's subvectors of endogenous and exogenous variables used for calibration. So far we have only been working within the deterministic framework of CGE models.

The stochastic extension to the model which we are about to consider consists of associating additive error terms (for the demand functions) and multiplicative error terms (for the production or similar functions) with the *subsystem* of equations in (8.7), as proposed by Mansur and Whalley (1984). Let the relation in (8.7) include a vector of additive disturbances $U$ :[14]

$$
Y^S = S\big(X^S,\, \gamma\big) + U \tag{8.9}
$$

where $U$ is a vector of random terms of the same dimension as $Y^S$, with any distribution which is known and can be simulated. In particular, the distribution of $U$ need not be normal.[15] With no loss of generality, we may assume that the expectation of $U$ is zero, and that the covariance is known and equal to $\Sigma$. It is thus clear that the deterministic framework has been abandoned, albeit within the context of calibration.

As in equation (8.7), but now including the random term, we can write:

$$
Y_0^S = S\big(X_0^S,\, \gamma\big) + U_0 \tag{8.10}
$$

where $U_0$ has the same distribution as $U$. As in the case of deterministic calibration, we derive:

$$
\gamma = \bar{H}_S\big(Y_0^S - U_0,\, X_0^S\big)\,. \tag{8.11}
$$

Consequently, equation (8.8) no longer obtains. It is only true when $U_0$ is a vector of zeros. Thus,

[14]Letting this vector enter the equation multiplicatively does not affect our results; see the formulation in Appendix A.

[15]The elements of $U$ may be degenerate at zero if, by their economic nature, the equations used in the deterministic calibration do not contain random disturbances.

in this stochastic context, the function $\bar{H}_S\left(Y_0^S,\, X_0^S\right)$ yields an estimator $\hat{\gamma}_0$ for $\gamma$ :

$$\hat{\gamma}_0 = \bar{H}_S\big(Y_0^S,\, X_0^S\big)\,. \tag{8.12}$$

This, combined with the definition of this estimator (8.7) yields:

$$Y_0^S = S\big(X_0^S,\, \hat{\gamma}_0\big)\,. \tag{8.13}$$

The goal of this step in the proposed procedure is to derive the scalar or vector relationships from equations (8.10) and (8.13) possibly after performing some algebraic transformations as required by certain equation structures), allowing us to solve for some or all of the elements of $U$ (or for some algebraic transformation of the vector $U$ or its elements). Since the distribution of $U$ is known, these transformations allow us to derive a pivotal function for $\gamma$. Using equations (8.10) and (8.13) we easily find the following:

$$W\big(X_0^S,\, \hat{\gamma}_0,\, \gamma\big) = S\big(X_0^S,\, \hat{\gamma}_0\big) - S\big(X_0^S,\, \gamma\big) = U_0\,. \tag{8.14}$$

Since the distribution of $U_0$ is known (by assumption), the left-hand side of expression (8.14) defines a pivotal function for the parameter $\gamma$. Moreover, in a calibration system like the one defined in equations (8.7) and (8.8) the *number of calibrated parameters* contained in $\gamma$ ($k$ in our case) is always *equal to the number of equations.* In other words, $k$ is the dimension of $\gamma$, $Y^S$, and $U$. Lau, commenting on Mansur and Whalley (1984), makes a similar remark. If $\Sigma$ is the covariance matrix of $U$, we may simulate the pivotal function $T(\gamma)$ as follows:

$$T(\gamma) = W\big(X_0^S,\, \hat{\gamma}_0,\, \gamma\big)'\Sigma^{-1}W\big(X_0^S,\, \hat{\gamma}_0,\, \gamma\big) = U'\Sigma^{-1}U\,. \tag{8.15}$$

We find $c_\alpha$ such that:

$$\mathsf{P}\left[T(\gamma) \le c_\alpha\right] = \mathsf{P}\left[U'\Sigma^{-1}U \le c_\alpha\right] = 1-\alpha\,. \tag{8.16}$$

Finally, the confidence region we seek for $\gamma$ is defined as:

$$\Gamma_\gamma = \left\{\gamma \in \mathbb{R}^k : T(\gamma) \le c_\alpha\right\}. \tag{8.17}$$

In practice, the appropriate critical point $c_\alpha$ may not be analytically computable. To obtain an exact confidence region, we may fall back on Monte-Carlo tests [Dwass (1957), Barnard (1963), Dufour and Kiviet (1996, 1998), Dufour and Khalaf (2001)]. By assumption, it is possible to generate $N$ independent and identically distributed representations, $U_1,\, \dots\,,\, U_N$, of the vector $U$ using Monte-Carlo techniques and, by extension, $N$ independent and identically distributed representations, $T_i = U_i'\Sigma^{-1}U_i,\ i = 1,\, \dots\,,\, N$, of the pivotal function, $T(\gamma)$. Thus, the variables $T(\gamma)\,,\ T_1,\, \dots\,,\, T_N$, are independent and identically distributed.

Now consider the functions

$$\hat{F}_N(x) = \frac{1}{N}\sum_{i=1}^{N} s(x - T_i), \quad \hat{q}_N(x) = \frac{N\hat{F}_N(x) + 1}{N+1}, \tag{8.18}$$

where $s(x) = 1$ if $x \geq 0$ and $s(x) = 0$ if $x < 0$. If we assume that the distribution of $T(\gamma)$ is continuous, we easily see that

$$\mathsf{P}\{\hat{q}_N[T(\gamma)] \leq 1-\alpha\} = \frac{I[(1-\alpha)(N+1)]}{N+1}, \quad \text{for } \alpha \in (0,\, 1)\,, \tag{8.19}$$

where $I[x]$ is the largest integer less than or equal to $x$. In particular, if $(1-\alpha)(N+1)$ is an integer, we have:

$$\mathsf{P}\{\hat{q}_N[T(\gamma)] \leq 1-\alpha\} = 1-\alpha\,, \quad \alpha \in (0,\, 1)\,. \tag{8.20}$$

It follows that the set

$$\Gamma_\gamma(N) = \big\{\gamma : \hat{q}_N[T(\gamma)] \leq 1-\alpha\big\}\,, \tag{8.21}$$

is a level $1-\alpha$ confidence region for $\gamma$.

So far we have assumed that the model contains calibrated, but not free, parameters. Now we shall consider the case in which both parameter types appear in the model. This amounts to combining a priori information on the free parameters with the distributions of random variables *used* to construct confidence intervals for the calibrated parameters. If we can condition on a point estimate of the free parameters, we revert to the case discussed earlier in this section since the conditioning eliminates the extrinsic uncertainty. However, if the two sources of uncertainty are jointly accounted for, the approach proposed for the case with no free parameter changes, but not fundamentally. In fact, alongside the equations used in the deterministic calibration, and which are now considered to contain disturbances which are either additive or multiplicative, we now add not a point estimate of the vector of free parameters nor a confidence region for this vector, but rather an estimator with a distribution that is known a priori. Such hypotheses are often made in sensitivity analysis of CGE models [see, for example, Harrison and Vinod (1992)].

For example, consider the case of constant elasticity of substitution or transformation functions, like the Armington function we examined in section 4. We let the function and its associated first-order condition contain two multiplicative errors in the following manner:

$$Q = B\big[\,\delta M^{(\sigma-1)/\sigma} + (1-\delta)\,D^{(\sigma-1)/\sigma}\,\big]^{(\sigma-1)/\sigma} e^{u}\,, \tag{8.22}$$

$$\frac{M}{D} = \left(\frac{\delta}{1-\delta}\right)^{\sigma}\left(\frac{p_D}{p_M}\right)^{\sigma} e^{v}\,, \tag{8.23}$$

where $u$ and $v$ are random variables. The vector $(u,\, v)$ need not have a normal distribution, nor is it required that $u$ and $v$ be independent. Moreover, we assume a distribution for the estimator $\hat{\sigma}$ of the free parameter $\sigma$. If we have reasons to suspect that the distribution of this parameter is not independent of the vector $(u,\, v)$, we need to consider the joint distribution of $(\hat{\sigma},\, u,\, v)$, and its covariance matrix must be estimated before we can perform simulations. It remains to find the pivotal function for $\delta$ and $B$ — not a negligible task from an analytical perspective. The distribution of this pivotal function will be related to that of the vector $(\hat{\sigma}, u, v)$.

## 9. Conclusion

In this paper we have formalized the concept of calibration in CGE models and developed two statistical methods for constructing confidence intervals for the calibrated parameters of these models. One is based on a projection technique which allows the construction of confidence sets for calibrated parameters. It greatly facilitates the construction of confidence regions for the endogenous variables of the model. After discussing numerical methods for implementing the approach developed, the latter was illustrated on a CES function (the Armington function) frequently used in CGE models.

The second method allows one to extend the usual *deterministic calibration* of CGE models by adding stochastic disturbances to the equations of the model used in the calibration process and then to construct corresponding confidence sets for calibrated parameters using simulation techniques. This method uses the classical concept of a pivotal function for a parameter. The general nature of this procedure allows it to apply to several cases that frequently occur in CGE models. We used a Cobb-Douglas production function to illustrate it. These two new methods of statistical inference in CGE models go part way to solving one of the most serious econometric problems associated with the calibrated parameters of the models and provide a way to manage the issue of uncertainty in the calibration of CGE models.

# A. Apppendix: Multiplicative disturbances

Generally in economics, equations are assigned multiplicative stochastic terms to assure non-negativity of the endogenous variables. We use this assertion in the proof. Let equation $(8.7)$ contain multiplicative errors disturbances as follows:

$$Y^s = U\, S\big(X^s,\, \gamma\big)\,, \tag{A.1}$$

or

$$Y^{s\prime} = S\,(X^s,\, \gamma)'\, U' \tag{A.2}$$

where $U$ is a square diagonal matrix whose dimension equals the number of elements of $Y^s$. The distribution of the elements of $U$ is known and can be simulated, it does not need to be normal.[16] With no loss of generality, we may assume that the expectation of the elements of $U$ is unity, and the known covariance matrix is denoted $\Sigma$. Considering our comment on the non-negativity of the endogenous variables, and thus of the random terms associated with the equations, (A.1) may be written:

$$\ln\,(Y_i^s) = \ln\,(U_i) + \ln\,[S_i\,(X^s,\, \gamma)]\,, \quad i = 1,\, \ldots\,,\, k. \tag{A.3}$$

With this form, if the disturbance terms were additive the logarithmic transformation would not be performed. With the appropriate changes in variables we revert to the additive disturbances dealt with in the text.

[16]An element of $U$ is set to $1$ when the corresponding equation does not involve a random disturbance.

# B. Apppendix: GAMS-MINOS program for the applications

```
$TITLE        PROJECTION METHOD APPLICATION
$TITLE        ARMINGTON-TYPE CES FUNCTION
OPTION NLP = MINOS5;
* A MODEL OF TYPE 1-2-3 AND A TWO SECTOR MODEL
* PARAMETERS DECLARATION
* THE REFERENCE-YEAR VALUES FOR THE CALIBRATION
* (OBTAINED FROM MOROCCAN SAM 1985 AND 1990)
PARAMETERS
PDO          PRICE OF DOMESTIC GOOD IN THE REFERENCE-YEAR
PWMO         INTERNATIONAL PRICE OF IMPORT IN THE REFERENCE-YEAR
EO           NOMINAL EXCHANGE RATE IN THE REFERENCE-YEAR (FOR CONVERSION)
PMO          DOMESTIC PRICE OF IMPORTED GOOD IN THE REFERENCE-YEAR
QO           DEMAND FOR THE COMPOSITE GOOD IN VOLUME IN THE REFERENCE-YEAR
MO           IMPORTS IN VOLUME IN THE REFERENCE-YEAR
DO           INTERNAL DEMAND FOR DOMESTIC GOOD IN THE REFERENCE-YEAR
TMO          TARIFF ON IMPORTS IN THE REFERENCE-YEAR
TAXMO        TAX ON IMPORTS IN THE REFERENCE-YEAR
;
VARIABLES
* IN THE PROJECTION APPROACH TO CONSTRUCT CONFIDENCE SETS FOR THE CALIBRATED
* PARAMETERS, SIGMA, DELTA AND BM ARE VARIABLES
* THEY ARE A PARAMETER IN STANDARD CGE MODEL
SIGMA        ELASTICITY OF SUBSTITUTION BETWEEN IMPORTED AND DOMESTIC GOODS
BM           SCALE PARAMETER IN THE CES FUNCTION.
DELTA        SHARE PARAMETER IN THE CES FUNCTION
OBJ          OBJECT VARIABLE IN THE OPTIMIZATION PROGRAM
;
* DATA AND CALCULUS
SCALAR
PDO          /1/
EO           /1/
PWMO         /1/
$ONTEXT
* REFERENCE-YEAR DATA FOR 1985
DO           /209847/
MO           /42806/
TAXMO        /9046.7/
$OFFTEXT
$ONTEXT
* REFERENCE-YEAR DATA FOR THE AGRICULTURAL SECTOR 1990
DO           /65341.32/
MO           /4248.00/
TAXMO        /-391.79/
$OFFTEXT
$ONTEXT
* REFERENCE-YEAR DATA FOR THE INDUSTRIAL SECTOR 1990
DO           /257868.02/
MO           /59327.9/
TAXMO        /10048.10/
$OFFTEXT
;
QO = MO + DO;
TMO = TAXMO / MO;
PMO = PWMO*(1+TMO)*EO;
DISPLAY DO, MO, QO, PWMO, EO, PDO, PMO, TMO, TAXMO;
* INITIALIZATION OF VARIABLES
$ONTEXT
```

```
* INITIALIZATION FOR ALL CASES EXCEPT MINIMIZING BM IN AGRICULTURE
SIGMA.L = 1.432371;
BM.L = 1.826;
DELTA.L = 0.285;
OBJ.L = 1.43;
$OFTEXT
$ONTEXT
*INITIALIZATION FOR MINIMIZING BM IN AGRICULTURE
SIGMA.L = 0.5;
BM.L = 1.001;
DELTA.L = 0.04;
OBJ.L = 1.470;
$OFTEXT
* BOUNDS ON VARIABLES
$ONTEXT
* FOR 1985
SIGMA.LO = 0.78381698;
SIGMA.UP = 2.080925014;
$OFFTEXT
$ONTEXT
* FOR AGRICULTURAL AND INDUSTRIAL SECTOR IN 1990
SIGMA.LO = 0.5;
SIGMA.UP = 4.5;
*$OFFTEXT
* EQUATIONS FOR MINIMIZING AND MAXIMIZING DELTA AND BM
EQUATIONS
UPSIGEQ          UPPER BOUND FOR SIGMA
LOSIGEQ          LOWER BOUND FOR SIGMA
DELTAEQ          CALCUL OF THE SHARE PARAMETER
BMEQ             CALCUL OF THE SCALE PARAMETER
OBJEQ            OBJECT FUNCTION
;
$ONTEXT
* FOR 1985
UPSIGEQ.. SIGMA =L= 2.080925014;
LOSIGEQ.. SIGMA =G= 0.783816986;
$OFFTEXT
$ONTEXT
* FOR AGRICULTURAL AND INDUSTRIAL SECTOR IN 1990
UPSIGEQ.. SIGMA =L= 4.5;
LOSIGEQ.. SIGMA =G= 0.5;
*$OFFTEXT
BMEQ..           QO =E= BM*(DELTA*MO**(-((1-SIGMA)/SIGMA))+
                    (1-DELTA)*DO**(-((1-SIGMA)/SIGMA)))**(SIGMA/(SIGMA-1));
DELTAEQ..        MO =E=(((DELTA/(1-DELTA))**(SIGMA))*((PDO/PMO)**SIGMA))*DO;
OBJEQ..          OBJ =E= DELTA;
OBJEQ..          OBJ =E= BM;
OPTIONS LIMROW = 0, LIMCOL = 0 ;
MODEL ARMIG /DELTAEQ, UPSIGEQ, LOSIGEQ, OBJEQ/;
MODEL ARMIG /DELTAEQ, BMEQ, UPSIGEQ, LOSIGEQ, OBJEQ/;
SOLVE ARMIG USING NLP MAXIMIZING OBJ;
SOLVE ARMIG USING NLP MINIMIZING OBJ;
DISPLAY SIGMA.L, DELTA.L;
DISPLAY SIGMA.L, DELTA.L, BM.L;
```

## References


Abdelkhalek, T. (2003), Libéralisation commerciale et pauvreté au Maroc: analyse à partir d'un modèle calculable d'équilibre général micro simulé, Technical Report 222, cahier de recherche de l'équipe MIMAP-Maroc, INSEA, Rabat, Morocco.

Abdelkhalek, T. and Dufour, J.-M. (1998), 'Statistical inference for computable general equilibrium models, with application to a model of the Moroccan economy', *Review of Economics and Statistics* **LXXX**, 520–534.

Abdelkhalek, T. and Martens, A. (1996), Macroclosures in computable general equilibrium models: A probabilistic treatment with application to Morocco, Technical Report 2596, Centre de recherche et développement en économique, Université de Montréal, Montreal.

Abrego, L. and Whalley, J. (2000), Decomposing wage inequality change using general equilibrium models, Technical report, International Monetary Fund and University of Western Ontario.

Barnard, G. A. (1963), 'Comment on 'The spectral analysis of point processes' by M. S. Bartlett', *Journal of the Royal Statistical Society, Series B* **25**, 294.

Bernheim, D. B., Scholz, J. K. and Shoven, J. B. (1989), Consumption taxation in a general equilibrium model: How reliable are simulation results?, Technical report, Department of Economics, Stanford University, Stanford, California.

Brooke, A., Kendrick, D. and Meeraus, A. (1988), *GAMS: A User's Guide*, The Scientific Press, Palo Alto, Redwood City, CA.

Byron, R. P. (1978), 'The estimation of large social account matrices', *Journal of the Royal Statistical Society, Series A, General* **141**, 359–367. Correction 142, 404.

Dawkins, C., Srinivasan, T. N. and Whalley, J. (2001), Calibration, *in* J. J. Heckman and E. E. Leamer, eds, 'Handbook of Econometrics, Volume 5', North-Holland, Amsterdam.

de Melo, J. and Robinson, S. (1989), 'Product differentiation and general equilibrium models of small economies', *Journal of International Economics* **27**, 47–67.

Decaluwé, B. and Martens, A. (1988), 'CGE modeling and developing economies: A concise empirical survey of 73 applications to 26 countries', *Journal of Policy Modeling* **10**, 529–568.

Devarajan, S., Lewis, J. D. and Robinson, S. (1986), A bibliography of computable general equilibrium (CGE) models applied to developing countries, Technical Report 224, Department of Economics, Harvard University, Cambridge, MA.

Devarajan, S., Lewis, J. D. and Robinson, S. (1990), 'Policy lessons from trade-focused two sector models', *Journal of Policy Modeling* **12**, 625–657.

Devarajan, S., Lewis, J. D. and Robinson, S. (1994), Getting the model right: The general equilibrium approach to adjustment policy, Technical report, Department of Economics, Harvard University, Cambridge, MA.

Dufour, J.-M. (1989), 'Nonlinear hypotheses, inequality restrictions, and non-nested hypotheses: Exact simultaneous tests in linear regressions', *Econometrica* **57**, 335–355.

Dufour, J.-M. (1990), 'Exact tests and confidence sets in linear regressions with autocorrelated errors', *Econometrica* **58**, 475–494.

Dufour, J.-M. (1997), 'Some impossibility theorems in econometrics, with applications to structural and dynamic models', *Econometrica* **65**, 1365–1389.

Dufour, J.-M. and Jasiak, J. (2001), 'Finite sample limited information inference methods for structural equations and models with generated regressors', *International Economic Review* **42**, 815–843.

Dufour, J.-M. and Khalaf, L. (2001), Monte Carlo test methods in econometrics, *in* B. Baltagi, ed., 'Companion to Theoretical Econometrics', Blackwell Companions to Contemporary Economics, Basil Blackwell, Oxford, U.K., chapter 23, pp. 494–519.

Dufour, J.-M. and Kiviet, J. F. (1996), 'Exact tests for structural change in first-order dynamic models', *Journal of Econometrics* **70**, 39–68.

Dufour, J.-M. and Kiviet, J. F. (1998), 'Exact inference methods for first-order autoregressive distributed lag models', *Econometrica* **66**, 79–104.

Dwass, M. (1957), 'Modified randomization tests for nonparametric hypotheses', *Annals of Mathematical Statistics* **28**, 181–187.

Gouriéroux, C. and Monfort, A. (1989), *Statistique et modèles économétriques, Volumes 1 et 2*, Economica, Paris.

Groupe de recherche en économie internationale (G.R.E.I.) (1992), La matrice de comptabilité sociale du Maroc de 1985, Technical report, Faculté des sciences juridiques, économiques et sociales, Centre d'études stratégiques, Université Mohammed V, Rabat.

Gunning, J. W. and Keyzer, M. (1995), Applied general equilibrium models for policy analysis, *in* J. Behrman and T. N. Srinivasan, eds, 'Handbook of Development Economics', Vol. 3A, North-Holland, Amsterdam, pp. 2026–2107.

Harrison, G. W. (1989), The sensitivity analysis of applied general equilibrium models: A comparison of methodologies, Technical report, Department of Economics, University of New Mexico.

Harrison, G. W., Jones, R., Kimbell, L. J. and Wigle, R. (1993), 'How robust is applied general equilibrium analysis?', *Journal of Policy Modeling* **15**, 99–115.

Harrison, G. W. and Vinod, H. D. (1992), 'The sensitivity analysis of applied general equilibrium models: Completely randomized factorial sampling designs', *Review of Economics and Statistics* **74**, 357–362.

Harrison, Glenn, W. (1986), A general equilibrium analysis of tarrif reduction, *in* (Srinivasan and Whalley 1986).

Jorgenson, D. W. (1984), Econometric methods for applied general equilibrium analysis (with comments), *in* (Scarf and Shoven 1984), pp. 139–205.

Kehoe, T. (1983), 'Regularity and index theory for economies with smooth production technologies', *Econometrica* **51**, 895–917.

Kiviet, J. F. and Dufour, J.-M. (1997), 'Exact tests in single equation autoregressive distributed lag models', *Journal of Econometrics* **80**, 325–353.

Manne, A. S. (1985), 'On the formulation and solution of economic equilibrium models', *Mathematical Programming Study* **23**, 1–22.

Mansur, A. and Whalley, J. (1984), Numerical specification of applied general equlibrium models: Estimation, calibration, and data, *in* (Scarf and Shoven 1984).

Martens, A. (1993), La politique économique de développement et les modèles calculables d'équilibre général: un mariage à la progéniture abondante, *in* D. Benali and A. Martens, eds, 'Collection du G.R.E.I., Volume 1', Université Mohamed V, Rabat, Morocco, pp. 96–119.

Pagan, A. R. and Shannon, J. (1985), Sensitivity analysis for linearized computable general equilibrium models, *in* J. Piggot and J. Whalley, eds, 'New Developments in Applied General Equilibrium Analysis', Cambridge University Press, Cambridge, U.K.

Pagan, Adrian, R. and Shannon, J. (1989), 'How reliable are ORANI conclusions?', *Economic Record* **63**, 33–45.

Rao, C. R. (1973), *Linear Statistical Inference and its Applications*, second edn, John Wiley & Sons, New York.

Reinert, K. and Roland-Holst, D. W. (1992), 'Armington elasticities for United States manufacturing sectors', *Journal of Policy Modeling* **15**, 631–639.

Scarf, H. E. and Shoven, J. B., eds (1984), *Applied General Equilibrium Analysis*, Cambridge University Press, Cambridge, U.K.

Shoven, J. B. and Whalley, J. (1984), 'Applied general equilibrium models of taxation and international trade: an introduction and survey', *Journal of Economic Literature* **22**, 1007–1051.

Shoven, J. B. and Whalley, J. (1992), *Applying General Equilibrium*, Cambridge University Press, Cambridge, U.K.

Srinivasan, T. N. and Whalley, J., eds (1986), *General Equilibrium Trade Policy Modelling*, The MIT Press, Cambridge, MA.

Wigle, R. (1986), Summary of the panel and floor discussion, *in* (Srinivasan and Whalley 1986), pp. 323–354.

Wigle, R. (1991), 'The Pagan-Shannon approximation: Unconditional systematic sensitivity analysis in minutes', *Empirical Economics* **16**, 35–49.